THE CONCEPT OF FREE WILL AS AN INFINITE METATHEORETIC RECURSION

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ABSTRACT

It is argued that the concept of free will, like the concept of truth in formal languages, requires a separation between an object level and a meta-level for being consistently defined. The Jamesian two-stage model, which deconstructs free will into the causally open "free" stage with its closure in the "will" stage, is implicitly a move in this direction. However, to avoid the dilemma of determinism, free will additionally requires an infinite regress of causal meta-stages, making free choice a hypertask. We use this model to define free will of the rationalist-compatibilist type. This is shown to provide a natural three-way distinction between quantum indeterminism, freedom and free will, applicable respectively to artificial intelligence (AI), animal agents and human agents. We propose that the causal hierarchy in our model corresponds to a hierarchy of Turing uncomputability. Possible neurobiological and behavioral tests to demonstrate free will experimentally are suggested. Ramifications of the model for physics, evolutionary biology, neuroscience, neuropathological medicine and moral philosophy are briefly outlined.

KEY WORDS

CLASSIFICATION

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INTRODUCTION

Free will (FW) is a concept in philosophy that refers to the putative capacity of a human agent to control her behavior by choices made by an act of will, on basis of her personal motives, convictions and intentions. The concept rests on the belief that human behavior is not fully determined by external causes. Moreover, her motives, convictions and intentions are themselves not determined by fully external causes, but self-determined. From a common sense perspective, we feel we are free in making decisions. Yet it continues to be debated, even after centuries of argumentation, how to coherently define free will and whether it exists in Nature [1].

FW can be regarded as freedom from some constraint: exactly what that constraint is, has remained moot. There are two broad philosophical positions on FW: *Incompatibilism*, which holds that the relevant constraint is determinism, and *Compatibilism*, which holds that determinism is irrelevant to the definition of free will, and that determinism and FW are compatible.

Two divergent Incompatibilist positions are *Hard Determinism*, which regards FW as false and determinism as true, and (metaphysical) *Libertarianism*, which regards determinism as false and FW as true. A FW *Skeptic* is an Incompatibilist who goes farther than a Hard Determinist and denies that FW is even a coherent concept. From the Skeptic perspective, indeterminism no more allows an agent control and self-determination over actions than does determinism. Since determinism and indeterminism are the only two logically possible causal primitives, this view holds that very concept of FW is meaningless. This Skeptic stand is sometimes called the dilemma of determinism or the standard modern argument against FW.

The libertarian may imagine that housed in her brain is an immaterial agency (such as a soul or homunculus) that somehow transcends the cause and effect law that holds elsewhere in Nature. This is a logically tenable defense of Libertarianism against Hard Determinism, even if an adherent of the latter would deny the existence of such immaterial agencies. But even the soul offers Libertarianism no protection against the Skeptic, because if the soul's choices transcend causality, they must be random, or results of random properties, again making the case for FW bleak!

The present work hopes to convince the FW Skeptic that a coherent metaphysical account of FW is possible. Of importance to our work is using an idea similar to that used by Tarski to define the concept of truth, except that here it is applied to causality. We think that FW is a causal primitive different from both determinism and indeterminism in being *metatheoretic*. Our attempt to define it shows it to be a form of causation that straddles endless levels of causality, *provided* we wish to form a coherent scientific narrative that fits in with out intuitive sense of self, freedom and responsibility.

This article is structured as follows. A version of the Jamesian two-stage model is proposed in Section 2, which would be useful for our further discussion. Although such a model has been considered as the defense of FW against the dilemma of determinism, we point out in Section 3 that the model fails, because the dilemma can be recursively resurrected. In Section 4, we show that an infinite recursion of Jamesian two-stage models restores a measure of protection for libertarianism against the dilemma. This model is adapted to define a Rationalist-Compatibilist FW. The connection of FW to uncomputability [2] and Tarskian undefinability for formal truth [3] are outlined in Section 5. We then discuss the consequences of the model for neurobiology in Section 6, before concluding in Section 7.

THE TWO-STAGE MODEL, 2*S*: A NEW HOPE

The two-stage model [4], introduced in its original form by W. James [5], and proposed in various forms by a number of researchers, is intended to defeat the dilemma of determinism.

The model posits that FW is a two-stage process: first there is freedom at the lower stage, and then there is will at the higher stage, which makes a choice. We present further in the text a version of it, which we call '2S', with several changes in the details, as described in the following two subsections.

PHYSICAL STAGE $L0$ (FREEDOM)

At the moment an agent's attention is drawn to a conflict situation, during a short time span, called the *selection window*, in a localized region of brain, which we refer to as the *free-will oracle*, probably in the pre-frontal cortex, the physical laws W_0 are put on a "causally open" mode. In preparation of entry into this mode, alternative options x_0, described by probability distribution P_0, are generated in the agent's brain. The physical laws W_0 only determine P_0, but do not entirely fix the eventual choice x_0. And the selection is not completely determined by the past history of the *physical* universe. At the close of the selection window, W_0 in FW oracle is re-set to the "causally closed" mode, and the choice x_0 that is available on the FW oracle's "register" at that moment is expressed as the agent's action.

If the agent makes the choice mechanically, without a focused exertion of her will, then there is no mental causation influencing the choice (as described below), and at the close of the selection window, a random x_0 is selected according to probability distribution P_0.

METAPHYSICAL STAGE $L1$ (WILL)

If the agent decides to exert her will, then her choice must not be random but instead reflect the desires, intentions and beliefs characteristic of the agent. These properties constitute her *cognitive private space*, and may not have a full representation in the physical level L_0. Thus the will produces generally a *deviation* from P_0.

A simple way to describe this situation mathematically is by:

$$x_0 = \lambda_1(P_0), \tag{1}$$

where λ_1 is the "will function" that encodes the properties of her cognitive private space. Function λ_1 takes note of P_0, but is *not* part of W_0, because if it were, then there would be causal closure in the physical, and hence no freedom at L_0. Instead, λ_1 is determined by the laws W_1, which extend W_0 to L_1.

The fact that the causal openness at W_0 is replaced by causal closure of W_1 in the consolidated system in $L_0 + L_1$ suggests that these two levels can be ordered in a *causal hierarchy*, which we represent by the expression: $L_1 < L_0$. The ordering expresses that L_1 causally precedes L_0.

2S AND THE DILEMMA OF DETERMINISM

In the absence of action by the will, the FW oracle can be considered as a *probabilistic* input/output machine. To underscore this, sometimes we will refer to such indeterministic behavior as *freedom-without-will*. We introduce a level $L_{0.5}$ as essentially L_0 equipped with a source of pure randomness. Instances of freedom-without-will have their causal closure in $L_0 + L_{0,5}$, but in this case there is no deviation from P_0. When the will is exerted, the random variable X_0 representing free choice deviates from P_0, and free will transcends physical causality.

The attempted defense of FW against the dilemma of determinism using model 2S would be: FW is not deterministic because of freedom at the physical stage. Nor is it random because the eventual choice is self-determined, being fixed by personal preferences via λ_1.

Though not the definitive word on FW, the model 2S brings the new insight that FW is a new *causal primitive*, different from determinism and indeterminism. While the latter two can be associated with a causally closed lawfulness, and thus defined on a single causal level (say L_0), FW cannot. More of this further in the text.

FREE WILL IN 2*S* AND FORMAL TRUTH

As we indicated in Ref. [6], the 2*S* feature of introducing the metaphysical level to define FW on the physical level parallels Tarski's use [3] of a metalanguage in order to consistently define the concept of arithmetic truth in an object language. Tarski showed that without careful separation of the two levels, one would end up with logical antinomies like the liar's paradox "This statement is false" (which is true iff it is false).

Similarly, if we fail to separate the "free" and "will" stages in free choice, we will err in reducing the choice to determinism or randomness. We believe that part of the difficulty in understanding the nature of FW is due to a lack of appreciation for this level separation.

With this in mind, we may consider L_0 as the object stage, and L_1 as the metastage. The free-willed agent with 2*S* structure will be designated F_1. A deterministic physical system, which is causally closed at level L_0 is designated F_0, while an indeterministic quantum physical system, which has L_0 freedom-without-will will be denoted $F_{0.5}$.

DILEMMA OF DETERMINISM REVISITED

Although the model 2*S* resolves the dilemma of determinism after a fashion, still the dilemma can be resurrected at level L_1. To see this, note that the two-stage agent F_1, taken as a whole, is deterministic, in view of Eq. (1), and must thus lack FW. This is just Schopenhauer's argument, who picturesquely said, "Man can do what he wants but he cannot will what he wants" [7], in his prize-winning essay in response to the challenge posed by the Royal Norwegian Academy of Sciences in 1839[1].

To reinstate free will, we allow the will function λ_1 to be freely chosen, i.e., we apply 2*S* to the selection of λ_1. We extend the FW oracle from the physical level L_0 to the metaphysical L_1. Within the selection window, the laws W_1 governing level $L_0 + L_1$ become momentarily causally open in the region of the FW oracle. We thus have *freedom of the will* at L_1 whereby the $L_0 + L_1$ (i.e., physical-metaphysical) history of the universe does not fix λ_1.

We introduce a metalevel L_2, where a higher-order will function λ_2 ("will free will"), provides causal closure, by deterministically selecting a particular λ_1. We can think of λ_2 as representing a deeper aspect of the agent's character or disposition and as determining the type of the will function λ_1 that she will select. Function λ_2 comes from a further interior layer of the cognitive private space of the agent. By adding an element of spontaneity and self-determination, it thwarts the selection of λ_1 from being modelled as an input/output process at level L_1. In the metatheoretic representation, this is tantamount to treating F_1 as the object system, and an L_2-aspect as the metasystem. A simple mathematical way to represent this is by:

$$\lambda_1 = \lambda_2(P_1), \tag{2}$$

where P_1 is a *probability function* encoding L_1-preferences according to laws W_1. If the higher-order will λ_2 is not exerted, then P_1 would describe the indeterministic selection of will λ_1. But if this "will free will" is exerted, then there will be deviations from P_1 in the selection of λ_1. Substituting Eq. (2) into Eq. (1), we obtain:

$$x_0 = \lambda_1(P_0) = [\lambda_2(P_1)](P_0) \equiv \lambda_2^*(P_1, P_0). \tag{3}$$

The second-order will λ_2 selects a first-order will λ_1 depending on P_1, and then λ_1 selects x_0. The extended causal hierarchy is $L_2 < L_1 < L_0$, whereby L_2 causally precedes L_1, which in turn precedes L_0.

Here precedence refers not just to chronology but to recursion depth in the agent's cognitive private space. It does not refer to an *earlier* event on the same causal level, but to *deeper* or higher-order cause. For example, if λ_1 inclines Alice to give Bob a gift, then λ_2 could be her character trait of wanting to help him. A yet deeper cause (λ_3) would be wanting for good to him or people she likes. And so on.

We denote by F_2 the type of agent for who there is causal closure at W_2, and by $F_{1.5}$ one for who the will function λ_1 is selected indeterministically. Although the existence of F_2 resolves the dilemma of determinism on L_1, still the dilemma can be resurrected at level L_2. This is evident seeing that the action of λ_2^* in Eq. (3) is deterministic.

To reinstate free will at L_2, we recursively apply $2S$. We make L_2-law W_2 at the FW oracle also causally open within the selection window. Let the freedom of λ_2 be described by probability function P_2, determined by W_2. Causal closure occurs at W_3 via higher-order FW λ_3 from an even deeper aspect (L_3-aspect) of the agent's cognitive private space. We have

$$\lambda_2 = \lambda_3(P_2), \tag{4}$$

Substituting Eq. (4) into Eq. (2), we obtain:

$$x_0 = [\lambda_2(P_1)](P_0) = [[\lambda_3(P_2)](P_1)](P_0) \equiv \lambda_3^*(P_2, P_1, P_0). \tag{5}$$

We extend the causal hierarchy as $L_3 < L_2 < L_1 < L_0$. An agent with causal closure of her choosing process in W_3 is denoted F_3 and one with freedom-without-will on L_2 (i.e., L_2-indeterministic) is denoted $F_{2.5}$.

Although the existence of F_3 appears to resolve the dilemma of determinism on stage L_2, still the dilemma can be resurrected at stage L_3 since F_3, in view of Eq. (5), can be considered as a deterministic system. This may be seen as a further higher-order extension of Schopenhauer's argument.

To prevent the dilemma of determinism at stage L_3, we introduce a $2S$ model on top of this stage, with F_3 as the object system, and L_4 as the metastage. But then the resultant F_4 will be deterministic. We require L_5 to obtain a $2S$ model for F_4 to avoid the dilemma of determinism at level L_4. Continuing this trend indefinitely, at level L_n, where n is any positive integer, we have:

$$\lambda_{n-1} = \lambda_n(P_{n-1}). \tag{6}$$

Substituting this recursively into lower levels into Eq. (3), we obtain:

$$x_0 = [[\cdots[[\lambda_n(P_{n-1})](P_{n-2})]\cdots](P_1)](P_0) \equiv \lambda_n^*(P_{n-1}, \cdots, P_0). \tag{7}$$

Evidently, we can still resurrect the dilemma of determinism at level L_{n+1} no matter how large n is, since the fact of λ_n^* in Eq. (7) is a deterministic function. Thus, the problem posed by this dilemma does not disappear for agent F_n but is merely postponed.

In response to this seemingly insurmountable difficulty posed by the recursive version of the dilemma of determinism, the only option to save libertarianism seems to be to let a free-willed agent be an infinite-stage entity, F_∞. What this means is that, for any *finite* integer n, the laws (W_n) of cause and effect at the stage L_n will lack causal closure in the region of the extended FW oracle during the selection window, and the selection of will λ_n cannot be modelled as a probabilistic or deterministic input/output system at stage. The causal closure for W_n will come through a higher-order cause λ_{n+1}, which is determined by the L_{n+1}-aspect of the cognitive private space. Therefore, the agent's choice of λ_n will transcend n^{th}-order causality W_n, so that the choice of λ_n should be regarded as spontaneous and self-determined at that stage.

In FW so understood, there is libertarian freedom in the sense that the agent's free choice has an inexhaustible causal depth in her cognitive private space. Perhaps, when we scan the inner space of our consciousness, and feel that our choices are free, it is this infinitude that we grasp intuitively, and feel inclined to report as genuine personal freedom.

Identifying human agents with F_∞ also means that FW is at least a *supertask* [8], a process that involves a sequence of countably infinite number of steps executed in finite time. In Section 5, we will present an argument suggesting that free choice is probably even a *hypertask*, which involves uncountably many steps executed in finite time.

FREE WILL AS AN INFINITE METATHEORETIC RECURSION

The free agent F_∞ in our model is an infinite-stage entity straddling the physical L_0 and the "final" or "infinite-th" stage, denoted L_∞. For simplicity, we will refer to the agents L_0-aspect as the "physical aspect" and the L_∞-aspect as "transfinite aspect". In a conflict situation, a response is initiated at the transfinite aspect and transmitted to the physical aspect, where it manifests as the choice x_0. It stands to reason that the final desination to which information about the sensory input is taken, before the agent's response is initiated, must also be the transfinite aspect.

AN INFINITE, STAGED CAUSATION

Extending (7), we can represent the choice x_0 through a sequence of downward causations starting from the "transfinite preference" P_∞:

$$x_0 = [[[\cdots[[[\lambda_\infty(P_\infty)]]\cdots]](P_2)](P_1)](P_0) \equiv \lambda^{\#}_n [P_n, P_{n-1},\cdots, P_\infty](P_{n-1},\cdots, P_0). \qquad (8)$$

The interpretation is that $\lambda^{\#}_n$, the will at stage L_n, is fixed by higher-order preferences, and then selects an outcome x_0 depending on lower-order preferences.

The form of Eq. (8) suggests that the larger the recursion depth n, the fewer the higher-order preferences P_{n+1}, $P_{n+2}, \cdots$ that could sway $\lambda^{\#}_n$ from the motivation encoded by P_∞. In Eq. (8), suppose that $0_{n-1}, \cdots, 0_0$ represent the probability functions $P_{n-1}, \cdots, P_0$ that are *unbiased* in the sense of being consistent with P_∞.

Replacing the lower-order preferences by their unbiased values in Eq. (8), we now define the nth-order *intent*

$$x_n = \lambda^{\#}_n [P_n, P_{n+1},\cdots, P_\infty](0_{n-1},\cdots, 0_0), \qquad (9)$$

meaning that x_n is the choice x_0 that *would* be made if there are no distortions downwards from level L_{n-1}. Thus we may call x_∞ as the "prime intent" or "transfinite intent", the option that would be selected if the will at infinity, λ_∞, were to act unthwarted on the physical.

During the act of free choice, the prime intent is replaced stage-wise by lowerorder intents, until the final choice is reached. We may refer to this infinite train

$$X \equiv x_\infty \to \cdots x_n \to x_{n-1} \to \cdots \to x_1 \to x_0, \qquad (10)$$

as the "descent of the will". This immediately evokes the notion of FW as the effectiveness of communication of x_∞ from the transfinite aspect to the physical aspect, undistorted by lower-order preferences. The will is stronger, if this channel of communication ("volition channel") is clearer, uncluttered by lower-order motivations, beliefs and desires inconsistent with their transfinite counterparts. This line of thought forms the basis of the compatibilist FW introduced further in the text.

RATIONALIST-COMPATIBILIST FREE WILL

The model above can be extended to protect FW from what may be called the 'rationalist/robot paradox'. By definition, a free, rational agent will, when faced with a choice, select the optimal option. His behavior is completely predictable, assuming that there is a single rational option. For a libertarian, rationality appears to undermine freedom [9]. Now this is not the case, as viewed by a compatibilist. But the rationalist/robot paradox asks how the compatibilist would differentiate a rational agent from an optimal robot programmed to choose rationally.

Following the line of thought indicated previously, we would like to think of the correlation between the prime intent and final choice as a measure of FW, since it expresses how well the agent is able to hold on to her prime intent by overcoming deviating influences. However,

this correlation would stay as merely incidental, unless the physical aspect holds x_∞ as her purpose or motive. For this, she must have cognizance of x_∞. Precisely this defines the role played by the agent's rational faculty or reasoning in FW. We will refer to an $\boldsymbol{F}_\infty$ agent equipped with the rational faculty by #$\boldsymbol{F}_\infty$.

The role played by reason is crucial. Without it, the physical aspect has no motive to deviate behavior from that determined by physical causality W_0. Now there may be random deviations from P_0 (applicable to animal agents), but they would be devoid of any systematic or deliberate attempt to transcend W_0. By contrast, a human agent, on the recommendation of reason, tries to overcome the imposition of W_0 by trying to deviate X_0 towards X_∞. Here the quantity X_j represents the random variable corresponding to x_j, i.e., values of variables associated with a probability distribution. Thus the reasoning faculty serves as the basis through which the opportunity provided by causal openness is exploited.

An agent is free to the extent that she is able to enforce her transfinite will on her physical choice. (Complications arising from the corruption of the rational faculty will be ignored here.) This gives us a quantification of Rationalist-compatibilist FW:

$$\boldsymbol{G} = \mathrm{Corr}(X_\infty : X_0), \tag{11}$$

where Corr is any measure normalized so that $-1 \leq \boldsymbol{G} \leq +1$. The rational free-willed agent is characterized by $\boldsymbol{G} = 1$, while a person completely under the sway of material nature, by $\boldsymbol{G} = -1$.

Lacking (substantial) reasoning, an animal may be represented simply by $\boldsymbol{F}_\infty$. The animal is free, but not free-willed. We express this insight with the expression:

$$\text{Freedom} + \text{reason} = \text{free will} \tag{12}$$

Quantum matter, or in particular quantum AI, which remains under the scope of physical causality, is a $\boldsymbol{F}_{0,5}$ agent, while a classical robot is a $\boldsymbol{F}_0$ agent. It is clear how this Rationalist-compatibilist account protects FW from rationalist/robot paradox: a deterministic robot is a $\boldsymbol{F}_0$ agent, while a rational free-willed agent is a #$\boldsymbol{F}_\infty$ agent with $\boldsymbol{G} = 1$.

The brain is arguably a special organ, whose physical structure has somehow been evolved equipping it with a FW oracle, providing a gateway to the transfinite aspect. AI lacks this and the physical laws governing its dynamics are causally closed.

It seems to be an interesting proposition that plants and "lower animals" (like microbes), which lack a central nervous system (CNS), could be considered as intermediary agents between quantum matter and higher animals (like mammals, reptiles and birds, which have a CNS), and thus represented by $\boldsymbol{F}_K$, where $0{,}5 < K < \infty$. Some of these ideas are summarized in Table 1.

Table 1. Freedom gives spontaneity, reason gives self-determination, and freedom with reason is free will. AI, lacking a FW oracle and being thus just a special configuration of quantum matter, is described by first-order indeterminism. By contrast, the higher animal or human brain, being equipped with a FW oracle, has freedom at all orders. Perhaps the $\boldsymbol{F}_\infty$ structure, common to humans and animals, is necessary for emotional behavior.

Entity	Agent type	Resource
Human	#$\boldsymbol{F}_\infty$	Free will (Freedom at all orders, plus reason)
Higher Animals (having a CNS)	$\boldsymbol{F}_\infty$	Freedom at all orders
Lower Animals and plants (Lacking a CNS)	$\boldsymbol{F}_K$ $(0{,}5 < K < \infty)$	Freedom up to a finite order
Quantum AI	$\boldsymbol{F}_{0,5}$	First-order freedom-without-will
Classical AI	$\boldsymbol{F}_0$	Determinism

CAUSAL VS. LOGICAL DETERMINISM

The resolution of the rationalist/robot paradox shows that the predictability of behavior does not imply that the behavior was causally determined (as in the robot's case). There is logical determinateness about the rational agent's behavior even though he transcends physical causality. We express this idea by:

$$\text{Logical determinism} \Rightarrow \text{Causal determinism}. \tag{13}$$

Now this result would be undermined if all humans were perfect ($G = 1$) then, even if $P_\infty \neq P_0$, we would be led to suspect that there is a "law of goodness", characterized by P_∞, that controls human behavior. However, some people are imperfect (having $G < 1$), suggesting that human behavior in general transcends causal determinism, and weakening the need to undermine the above conclusion.

EXPERIMENTAL TEST

It is an interesting and old[1] question: how to experimentally demonstrate the existence of FW? Our model suggests that unfocussed or casual acts of choice would be governed by a probability distribution P_0, while a free-willed action with deliberate intent will in general produce a deviation from P_0 towards P_∞.

The observation of discrepancy between the statistics of focused and unfocused choice, could be one way to demonstrate the existence of FW. Designing such an experiment may not be easy, since the very act of focusing may psychologically alter P_0, so that an observed deviation may either be due to will-induced deviation or due to an alteration of P_0 or due to both.

WHY WE ARE NATURAL LIBERTARIANS

The model also helps make sense of people's instinctive inclination to Libertarianism. It is a reasonable assumption that as an agent introspectively scans the inner space of her consciousness, depending on how subtle her awareness is, she can at best objectively perceive only so deep as there is freedom.

The unfreedom and higher-order causal influences lying beyond that point become part of her subjective consciousness, and she is unable to consciously experience them, though she may deduce them by observing her conscious choices and preferences.

FREE CHOICE AND UNCOMPUTABILITY

We now explain a line of thought indicated in Ref. [6], on the correspondence between the causal hierarchy and the hierarchy of Turing uncomputability, in reversed ordering. That is, given a $\#F_\infty$ agent, x_∞ is computable, while x_m is harder to compute than x_n if $m < n$.

The basic idea behind this claim is the following. Suppose one has a computer program (or Turing machine) so powerful that it can compute the free choice of a free-willed agent, using the current most detailed description of her brain state. Now if its prediction were shown to the agent, being free-willed, she may contradict the prediction. The conclusion is that such a powerful computer program does not exist. We now consider a somewhat more detailed argument.

Given a free-willed $\#F_\infty$ agent **A**, suppose there is a computer $\mathbf{T}_C$, programmable in some computer language $\mathbf{T}_L$ and suitable for the task of computing **A**'s free choice x_0. Let Â denote the description of **A** as a computer program in $\mathbf{T}_L$. We assume that all computer programs that encode in $\mathbf{T}_L$ the description of free-willed agents are denumerable and that Â is the ath program. Similarly, one is assumed to be able to encode situation any conflict situation **J** in the medium of $\mathbf{T}_L$ by a description Ĵ, and enumerate them alphabetically as some number j.

If free choice is computable, then the computer $\mathbf{T}_C$ can, given the enumerations for the $\mathbf{T}_L$ description of the agent and the conflict situation, compute the agent's free choice in finite time, or:

$$\tau_C(a; j) = \begin{cases} 0 \Leftrightarrow \mathbf{A}(\mathbf{J}) = 0, \\ 1 \Leftrightarrow \mathbf{A}(\mathbf{J}) = 1. \end{cases}, \tag{14}$$

where for simplicity we have assumed the outcome to be two-valued. (There is no loss of generality, since any computable output can be made binary, for example by assigning "0" if the outcome is non-numerical or numerical and less than 0, and "1" otherwise). In words, the computer produces output 1 (resp., 0) if $\mathbf{A}$ freely chooses 1 (resp., 0) when faced with conflict situation $\mathbf{J}$.

For any positive integer j, using this as a subroutine, we can build another program:

$$\tau_R(j) = \begin{cases} 1 \Leftrightarrow \tau_C(j; j) = 0, \\ 0 \Leftrightarrow \tau_C(j; j) = 1. \end{cases} \tag{15}$$

which is a representation of the above notion of the uncooperative free-willed agent. Thus $\mathbf{T}_R$ outputs "0" on input j iff the jth free-willed agent outputs "1" on the jth conflict situation.

We can now apply the computer to $\mathbf{T}_R$, so that from Eqs. (14) and (15):

$$\tau_C(t_R; j) = \begin{cases} 1 \Leftrightarrow \tau_C(j; j) = 0, \\ 0 \Leftrightarrow \tau_C(j; j) = 1. \end{cases}, \tag{16}$$

where t_R is the enumeration of $\mathbf{T}_R$. If we set $j := t_R$, then we are led to a contradiction as it would entail that $\mathbf{T}_C(t_R, t_R) = 0$ if and only if $\mathbf{T}_C(t_R, t_R) = 1$.

To restore consistency, we infer that $\mathbf{T}_C$ will never halt on inputs (t_R, t_R), which thereby constitutes an undecidable Gödel sentence under the above encoding. We conclude that in general x_0 will be uncomputable for the family of computer programs considered.

One can conceive a higher-level "meta-computer" program $\mathbf{T}_C[(1)]$ that is able to decide whether $\mathbf{T}_R(t_R)$ equals 0 or 1, but that is not contained in this family. If the cardinality of such meta-computers is greater than $\aleph_0$ (countable infinity), then the above diagonal argument based paradox can be averted, because the meta-computers will not be denumerable. This situation is similar to that pertaining to the concept truth, of requiring a metalanguage in order to define truth in the object language [3].

The proof given above for the uncomputability of x_0 is similar to the that of the uncomputability of the halting problem for Turing machines [2]. The concept of a meta-computer alluded to above indicates that the proof of uncomputability "relativizes", meaning that one can construct harder problems, by allowing a computer program to call as subroutine an "oracle" that solves the above free choice problem in bounded time. One can then construct a Gödel sentence for this oracle-enhanced program, which then yields a problem with its hardness shifted one level higher than the free choice problem above. Problems on the same level of uncomputability, i.e., uncomputable problems which are Turing-equivalent, form a Turing degree. The process can be repeated to construct higher Turing degrees, i.e., the next higher levels of more uncomputable problem [10]. It is known that there are $2^{\aleph_0}$ (uncountably many) Turing degrees.

We suggest that the the causal hierarchy corresponds to Turing degrees, but in inverse ordering, whereby the prime intent, which arises beyond an infinite number of causal stages, is itself Turing-computable. But the descent of the will would correspond to transition to higher levels of Turing uncomputability, making the free choice of agents of sufficiently low FW highly uncomputable. The act of FW in general must be a monstrous hypertask, since the lower causal stages correspond to ever higher orders of uncomputability.

Why should the causal and computational hierarchy correspond with each other? Here we will appeal to a teleological argument: that if the consequences of the causal hierarchy were computable, then there would have been no need for the "brute force" computation provided by the physical manifestation of the universe and human agents!

Considerable research has been devoted in computability theory and mathematical logic to the study of the complicated structure of Turing degrees. Perhaps all of that may have a bearing on the cognitive structure of free-willed agents.

NEUROLOGICAL BASIS FOR FW

The presence of the FW oracle in the human brain marks the basic difference between a human agent and a robotic simulation. The question of how the FW oracle is embedded in the brain and called forth, is briefly considered here.

A $\#F_\infty$ agent is an infinite entity, whereas the physical brain of a human being is finite in terms of its information storage and computation capacity. Therefore, if humans are $\#F_\infty$ agents, sufficiently high levels L_j cannot have a physical representation, i.e., a neural correlate.

We propose the following physical realization of the model. At the instance a human becomes aware of a choice, she is instinctively driven to enact her "nature", encoded by P_0. At the selection window, her "reason", which carries a representation of P_∞, advices her to deviate X_0 towards X_∞. This creates a potential tension, which may result in a fleeting quantum superposition, and may correlate with the agent subjectively experiencing an internal conflict.

Since P_0 is determined by W_0, its neural correlate is expected to be well defined, and associated with the motor cortex. Since P_∞ is largely computable, its neural correlate is also expected to be well defined, and associated with the reasoning circuits in the pre-frontal cortex. In Figure 1, these two correlates are represented as the slow and fast neural pathways, at whose confluence the FW oracle lies.
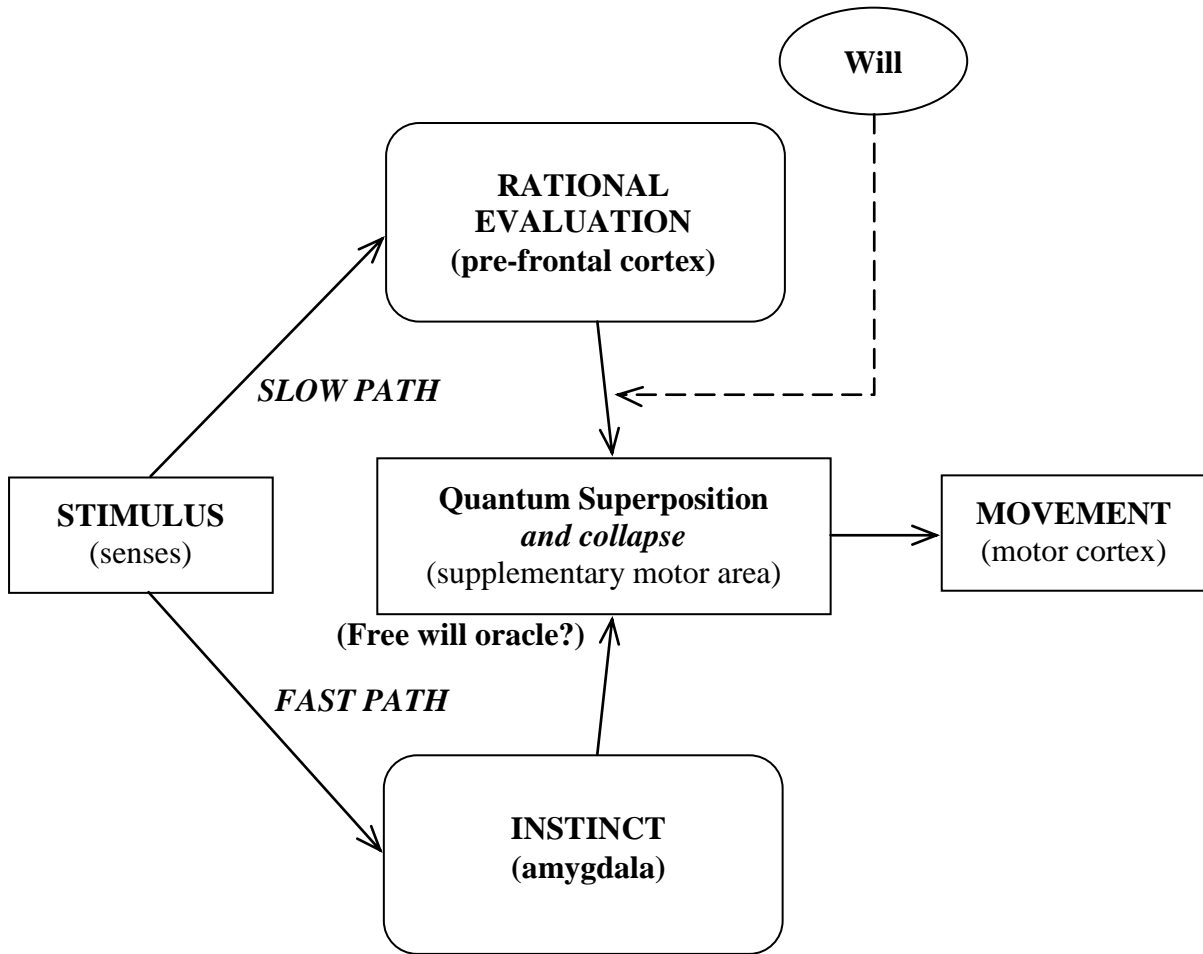
From physically observable data, one may be able to predict a pattern of behavior. However, since the higher levels in the causal hierarchy are not observable, therefore in any given instance of an agent's free choice, even the most detailed neural imaging (say via fMRI) will be unable in principle to predict with full certainty what the agent will select.

The FW mechanism in the brain of a $\#F_\infty$ agent can never be modelled as a finite input-output device. An important physical consequence is that, the mental causation that produces a deviation from W_0 may correspond to deviations from physical laws associated with W_0, like energy conservation or the Second Law of thermodynamics. For example, the initial voltage fluctuation in a motor neuron that initiates the spontaneous movement of a mouse's whisker, may be energy-wise unaccounted physically, even though the subsequent nonlinear amplification of that fluctuation to a physical action will certainly be governed by (classical) physics.

A possible experimental test of the model could aim to distinguish willful from casual choice. In the latter case, the probability of choice will reflect P_0, which can be estimated from the relative strengths of signals, as picked up by fMRI scans. Under willful choice, there will be a deviation in the probability of choice away from P_0 towards P_∞.

DISCUSSIONS AND CONCLUSIONS

We showed that, although metaphysical libertarianism is provisionally protected against the dilemma of determinism by the two-stage model, it is vulnerable to the recursive version of the dilemma. As a defense for libertarian freedom, we proposed the infinitely recursive two-stage scheme. A free agent is described here by F_∞, and free-willed agents by $\#F_\infty$. The concept of FW here is of the Rationalist-compatibilist kind.

Figure 1. Neurobiological flowchart for free choice by an agent. The conflict situation triggers neural signals along two paths: the fast "nature" path and the slow "reason" path-carrying possibly opposing recommendations for action. If the neural signal in the slow path is weak, the agent executes the instinctive action induced by the fast path. But if the signal in the slow path is sufficiently strong, and there is a conflict between P_0 and P_∞, then a quantum superposition is set up at the FW oracle, as the intermediate step for deviating X_0 towards X_∞.

Some other issues, with ramifications for quantum physics, neuroscience, mathematics, philosophy, computation theory, are briefly mentioned below.

There appears to be a parallelism between the will in F_1 and hidden variables in ontological models of quantum mechanics [6]. But there are two basic differences: these ontological models attempt *explain* probabilistic physical laws W_0 (and thus correspond to $F_{0,5}$), whereas the will in F_1 may produce *deviations* from W_0. Second, the will in F_1 is the first rung in an infinite hierarchy of higher-order willings, whereas hidden variable models of quantum mechanics stop at unit depth.

Neurobiology is the area most affected by our model, and also its possible clearest testing ground. Experimentally locating the seat of the FW oracle in the brain, and working out how mental causation initiates free-willed action in motor neurons will be vital. Experiments that distinguish willful and casual choice offer another window of study the neurobiological circuitry for FW.

This understanding can be medically useful. By potentially clarifying the roles neurotransmitters or receptors play in the process of free choice, it may be able to suggest

medical solutions that help encourage self-controlled behavior, by enhancing the "reason" pathway, rather than momentarily suppressing the "nature" pathway (Figure 1). Such treatments may be useful for patients suffering from neuropathological ailments like obsessive-compulsive disorder (OCD).

In mathematics and computation theory, the relationship between the causal hierarchy and Turing degrees would merit further study. This will help to elucidate the scope of AI. The formalization of FW as presented here, along the lines of formalization of the concept of truth [3] would be the first step here.

Our model implies that high FW correlates with improved predictability, i.e., reduced entropy in X_0. This reduction comes not by a compensatory increase in entropy elsewhere in the universe as required by the Second Law of thermodynamics, but by means of the deviation from W_0 produced by mental causation. Departure of P_j from 0_j is expressed as a conflict, and thus entropy, on L_0. A purely physical means to reduce entropy would be subject to the Second Law, with no implications for the agent's cognitive freedom. But by freely reducing this entropy, the agent is aligning her P_j's with 0_j's, and enhancing her freedom. Thus concepts like moral responsibility and justice are helpful as props that encourage free-willed behavior, and thereby help reduce disorder, though not necessarily on the physical level.

This in turn has implications for evolutionary biology. It suggests that the underlying force driving evolution was perhaps not Nature's quest for propagating the species most successful at survival, but instead Nature's quest for greater freedom. Darwin-like incremental evolution evolves quantum matter ($F_{0,5}$ agent) through lower animals lacking a CNS (F_J with finite J), and then through higher animals equipped with a CNS but no cerebral cortex (free agents, F_∞), and from there, finally to free-willed agents, $\#F_\infty$. Thus *Homo sapiens sapiens* perhaps already represents the limits of Darwinian biological evolution. The remaining evolutionary journey, towards greater freedom of the will, is now "up to us". It should be accomplished through self-determination.

REMARK

[1]The Royal Norwegian Academy of Sciences (KNVA), the predecessor of DNVA, posed in 1839 the academic question "Is it possible to demonstrate human free will and self consciousness?"

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