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SHORT RUN PROFIT MAXIMIZATION IN A CONVEX ANALYSIS FRAMEWORK

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ABSTRACT

In this article we analyse the short run profit maximization problem in a convex analysis framework. The goal is to apply the results of convex analysis due to unique structure of microeconomic phenomena on the known short run profit maximization problem where the results from convex analysis are deductively applied. In the primal optimization model the technology in the short run is represented by the short run production function and the normalized profit function, which expresses profit in the output units, is derived. In this approach the choice variable is the labour quantity. Alternatively, technology is represented by the real variable cost function, where costs are expressed in the labour units, and the normalized profit function is derived, this time expressing profit in the labour units. The choice variable in this approach is the quantity of production. The emphasis in these two perspectives of the primal approach is given to the first order necessary conditions of both models which are the consequence of enveloping the closed convex set describing technology with its tangents. The dual model includes starting from the normalized profit function and recovering the production function, and alternatively the real variable cost function. In the first perspective of the dual approach the choice variable is the real wage, and in the second it is the real product price expressed in the labour units. It is shown that the change of variables into parameters and parameters into variables leads to both optimization models which give the same system of labour demand and product supply functions and their inverses. By deductively applying the results of convex analysis the comparative statics results are derived describing the firm's behaviour in the short run.

KEY WORDS

short run profit maximization, duality, normalized profit function, Hotelling's lemma and its dual, comparative static analysis

CLASSIFICATION

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INTRODUCTION

The basic behavioural assumption in economics is that economic agents optimize subject to constraint. In the optimization problems convex sets take an important role in describing economic laws in almost every area of microeconomic theory. The possibility of describing convex sets in two ways leads to duality in microeconomic theory which can be defined as derivation and recovering of the alternative representations of consumer preferences and production technology [1-5].

The goal of the article is to apply the results of convex analysis due to unique structure of microeconomic phenomena on the known short run profit maximization problem where the results from convex analysis are deductively applied. This article expands our research of duality between the short run profit and production function [6]. In the primal optimization model the technology in the short run is represented by the short run production function and the normalized profit function, which expresses profit in the output units, is derived. In this approach the choice variable is the labour quantity. Alternatively, technology is represented by the real variable cost function, where costs are expressed in the labour units, and the normalized profit function is derived, this time expressing profit in the labour units. The choice variable in this approach is the quantity of production. The emphasis in these two perspectives of the primal approach is given to the first order necessary conditions of both models which are the consequence of enveloping the closed convex set describing technology with its tangents. The dual model includes starting from the normalized profit function and recovering the production function, and alternatively the real variable cost function. In the first perspective of the dual approach the choice variable is the real wage, and in the second it is the real product price expressed in the labour units. It is shown that the change of variables into parameters and parameters into variables leads to both optimization models which give the same system of labour demand and product supply functions and their inverses. By deductively applying the results of convex analysis the comparative statics results are derived describing the firm's behaviour in the short run.

The word duality comes in the economic literature for the first time in the work of Hotelling in 1932 who recognized that with the utility function (profit function) whose arguments are quantities, and whose derivatives are prices, there exists dually a function of prices whose derivatives are quantities (price potential) [7]. Jorgenson and Lau interpreted Hotelling's profit function as the production function, and price potential as the normalized profit function [8, 9]. The advantages of duality are especially recognized from an empirical standpoint, because the supply and demand functions are obtained by simple differentiation of the value functions which satisfy certain regularity conditions instead of solving the whole optimization problem. The second advantage of duality theory lies in the elegant comparative statics analysis which is implied by the properties of the value functions [4].

McFadden first proved McFadden duality theorem between the profit and production function [10]. From a theoretical point of view, after the recognition of the practical advantages of duality in microeconomic theory, authors were proving the duality theorems between various primal and dual functions starting from the various regularity conditions [4]. From an empirical point of view, technology parameters were estimated starting from the various functional forms of the dual functions, including the profit function. An alternative approach includes nonparametric estimation [11-15].

There exists a lot of literature devoted to the analysis of duality theory in empirical application. The most interesting question in this context is whether the estimates obtained in the primal approach are consistnt with those obtained in the dual approach [17, 18].

The remainder of the article is organized as follows. The next section analyses the short run profit maximization model from two perspectives, where in the first perspective the normalized profit function is derived by starting from the production function, and in the second perspective the normalized profit function is derived by starting from the real variable cost function. The third section includes recovering the production function and the real variable cost function from the normalized profit function and derivation of the comparative statics results. The fourth section gives an illustrative example of the results and the final section summarizes the obtained findings.

THE SHORT RUN PROFIT MAXIMIZATION PROBLEM

The basis for the application of duality theory in microeconomics is the price taking behaviour [1]. In this article we start from the perfectly competitive firm in the output and input market and analyse its behaviour in the short run. The starting point is the description of technology in the short run which is in the first approach described by the production function $y = f(L,\bar{K})$, where y is the output quantity, L is the labour quantity which is the variable input and $\bar{K}$ is the quantity of capital, which is fixed input in the short run. The choice variables of the perfectly competitive firm in the short run are the profit maximizing labour and output quantities.

Since the optimal variable input and output quantities are not influenced by the quantity of the fixed input, the short run profit function will be defined below as the difference between total revenue and variable cost,

$$\pi(p,w,\bar{K}) = \max_{L}\ pf(L,K) - wL\,, \tag{1}$$

where p is the product price and w is the price of the variable input. By dividing all prices in the model by the product price and expressing them in the units of product, the upper model reduces to the following equivalent model:

$$\frac{\pi\left(\frac{w}{p},\bar{K}\right)}{p} = \max_{L}\ f(L,\bar{K}) - \frac{w}{p}L\,. \tag{2}$$

The optimal value function in this optimization model is called the normalized profit function after Jorgenson and Lau [9]. It is the function that gives maximum profit in the short run expressed in the units of output. The firm chooses the quantity of variable input taking into account the quantity of the fixed input and the real wage. Therefore, the solution of the above optimization problem includes the variable input demand function, or the labour demand function, the supply function and the maximum short run normalized profit function.

By differentiating the goal function in (2) with respect to L, and for the given real wage $\left(\frac{w}{p}\right)^0$, the first order necessary condition is obtained, which implies that the firm will hire the level of labour for which the real wage is equal to the marginal product of labour,

$$\left(\frac{w}{p}\right)^0 = \frac{\partial f(L,\bar{K})}{\partial L}\,. \tag{3}$$

This is a known result in the microeconomic theory. The second order sufficient conditions imply decreasing marginal product of labour [16, 19],

$$f_{LL} < 0. \tag{4}$$

The goal is to apply the results of convex analysis on this short run profit maximization problem and to confirm the derived results graphically by enveloping the closed convex production set with its tangents.

Technology in the short run is represented by the production curve which expresses maximum output quantity that can be produced given fixed input quantity and the given technology. It is assumed that the production function is differentiable on its domain which implies that the production curve has the unique tangent in each point. This assumption is not necessary for the analysis and all results can be derived by not relying upon the differentiability assumption. We also assume that the production function is concave or that the production process in the short run is characterized by the diminishing marginal product of labour.

Let us look at the Figure 1 and let us choose some arbitrary labour quantity L^0 that, together with fixed capital in the short run, produces the output level $y^0 = f(L^0, \overline{K})$, which is represented by the point (L^0, y^0) on the production curve. If we draw a tangent on the production curve at this point, the slope of the tangent is the value of the marginal product of labour at this point, $\frac{\partial f(L)}{\partial L}(L^0) = f_L(L^0)$, and the equation of the tangent is

$$y = y^0 + f_L(L^0)(L - L^0). \tag{5}$$

We can look at the tangent from another perspective and give to it another interpretation. The first step is to interpret the production curve as the real revenue curve, expressing revenue in the product units. Real revenue is obtained by dividing total revenue with the product price. The next step is to include the graph of the real variable costs whose equation is $y = \left(\frac{w}{p}\right)^0 L$.

Real variable costs are costs are expressed in the units of output. They are represented by the line with the slope equal to the real wage. The goal is to find the labour level which makes the difference between the real revenue and the real variable costs, which is the real or normalized profit, the biggest as in (2).

The real variable cost curve can be interpreted as the isoprofit curve which gives all the combinations of labour and production for which the normalized profit is equal to zero. Generally, the equation of the isoprofit line is

$$y = \frac{\Pi}{p} + \left(\frac{w}{p}\right)^0 L \tag{6}$$

and for $\frac{\Pi}{p} = 0$ it collapses to $y = \left(\frac{w}{p}\right)^0 L$. From the equation in (6) it can be concluded that the normalized profit is graphically represented as the intercept of the isoprofit line. Therefore, we move isoprofit lines up until the tangency of the production curve and the isoprofit line is reached. For this level of labour the normalized profit is maximized and the isoprofit line is the tangent on the production curve. This implies that the real wage is equal to the marginal product of labour, which was already derived in (3). By solving the equation in (3) the labour demand function is obtained and the short run supply function is derived by inserting the labour demand function in the short run production function.

The equation of the isoprofit line which represents maximum profit and which is the tangent on the production curve is

$$y = \frac{\pi}{p} + \left(\frac{w}{p}\right)^0 L \,. \tag{7}$$

The comparative statics analysis require answering the question about the influence of the parameter change on the optimal solution, which in our context includes answering the question about the influence of the real wage change on the optimal labour and output levels, and finally on the normalized profit. In answering this question we will envelop the production curve with more tangents. So let's assume that the real wage increases, $\left(\frac{w}{p}\right)^1 > \left(\frac{w}{p}\right)^0$. For the given technology and fixed factor of production, the active producer will adjust to the new real wage change and hire less labour. This will lead to decreased production. Let's analyse new isoprofit line

$$y = \frac{\pi}{p} + \left(\frac{w}{p}\right)^1 L \tag{8}$$

determined by the new real wage and look for profit maximizing labour level. It is determined by the point of tangency between the production curve and the new isoprofit line. At the real wage $\left(\frac{w}{p}\right)^0$ the optimal labour level is L^0. The equation of a tangent on the production curve at $\left(\frac{w}{p}\right)^0$ is $y = \left(\frac{w}{p}\right)^0 L + \left(\frac{\pi}{p}\right)^0$ or equivalently, the equation of a line passing through the point $\left[L^0, f(L^0)\right]$ with the given slope $\left(\frac{w}{p}\right)^0$ is

$$y - f(L^0) = \left(\frac{w}{p}\right)^0 (L - L^0) \,. \tag{9}$$

At the real wage $\left(\frac{w}{p}\right)^1$ the optimal labour level is L^1. The equation of a tangent on the production curve at $\left(\frac{w}{p}\right)^1$ is $y = \left(\frac{w}{p}\right)^1 L + \left(\frac{\pi}{p}\right)^1$ or equivalently, the equation of a line passing through the point $\left[L^1, f(L^1)\right]$ with the given slope $\left(\frac{w}{p}\right)^1$ is

$$y - f(L^1) = \left(\frac{w}{p}\right)^1 (L - L^1) \,. \tag{10}$$

Since the graph of the concave production function is below its tangent, the following inequalities for the two labour levels hold: $f(L^1) \le f(L^0) + \left(\frac{w}{p}\right)^0 (L^1 - L^0)$

and $f(L^0) \le f(L^1) + \left(\frac{w}{p}\right)^1 (L^0 - L^1)$. From these two previous inequalities the important comparative statics result from the production theory follows,

$$(L^0 - L^1)\left[\left(\frac{w}{p}\right)^0 - \left(\frac{w}{p}\right)^1\right] \le 0. \tag{11}$$

It implies that an increase in real wage decreases demand for labour of the profit maximizing firm for the given technology and fixed input. This result was obtained by enveloping the closed convex set with its tangents, or by changing real wage. Intercepts of tangents on the vertical axis represent the value of maximum normalized profit for various values of real wages and for the given level of the capital.

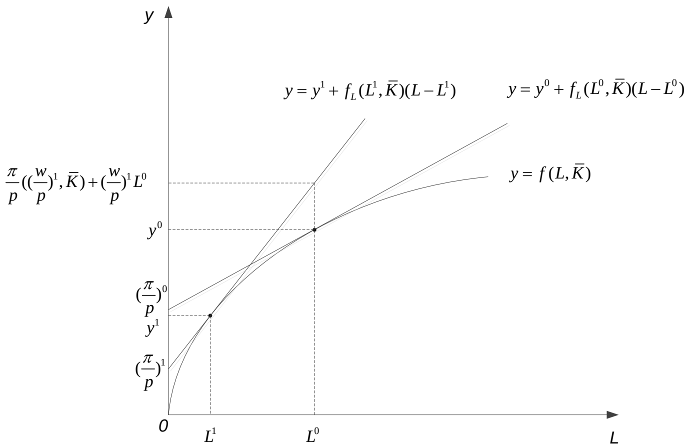

Figure 1. Deriving the Normalized Profit Function.

An alternative approach in deriving the normalized profit function includes starting from the variable cost function as representation of the firm's technology, $VC(y) = wL(y)$, where $L(y)$ represents minimum labour quantity for each output quantity and it is obtained by inverting the production function in the short run. Conditionally, the variable cost function gives minimum variable costs necessary to produce every production quantity. The short run profit maximization problem in this approach includes maximizing the difference between revenue and variable costs, where the decision variable is the quantity of production,

$$\pi(w, p, \bar{K}) = \max_y py - wL(y) \tag{12}$$

To analyse a profit maximization problem from this perspective in the convex analysis framework, we divide all prices in the model by the wage. So we are expressing the prices and profit in the labour units. The upper model consequently reduces to the following equivalent model:

$$\frac{\pi}{w}(\frac{p}{w}, \bar{K}) = \max_y \frac{p}{w} y - L(y). \tag{13}$$

In this approach profit is expressed in the units of labour and we are looking for the output quantity that gives the biggest difference between the revenue expressed in the units of labour and the variable costs expressed in the units of labour. By differentiating the goal function with respect to the output quantity, the first order necessary condition is obtained,

$$\frac{p}{w} = \frac{dL(y)}{dy} \tag{14}$$

or

$$p = w\frac{dL(y)}{dy} \tag{15}$$

which gives us the known microeconomic condition according to which the producer will supply the production quantity for which the marginal revenue, which is the price, is equal to the marginal cost, $w\frac{dL(y)}{dy}$.

To give this short run profit maximization model a convex analysis framework, let's conduct the same kind of analysis as before. The real variable cost curve is graphically represented in the space where the product quantity is measured on the horizontal axis and real variable costs are measured on the vertical axis. Since we started from the concave production function, the real variable cost function is convex in y [16, 19]. Next we add the graph of the real revenue whose equation is $L = \frac{p}{w}y$, which represents revenue in the labour units. It is the line with the slope equal to the real product price, or the product price expressed in the labour units. Graphically we are looking for the output quantity which gives the biggest difference between the real revenue and the real variable costs. The real revenue line can be interpreted as the isoprofit line for the level of normalized profit equal to zero, $L = \frac{p}{w}y - \pi$, which for $\pi = 0$, collapses to $L = \frac{p}{w}y$. Since our interest is in finding the maximum profit, we will move the isoprofit line up until the tangency of the cost curve and the isoprofit line is reached. For this level of quantity isoprofit line is the tangent on the cost curve and the slope of the isoprofit line is equal to the slope of the cost curve, or the product price is equal to the marginal cost. This result is also contained in the first order necessary condition for the problem of normalized profit maximization.

Let's assume now that the real product price increases, $\left(\frac{p}{w}\right)^1 > \left(\frac{p}{w}\right)^0$. For the given technology and fixed factor of production, the active producer will adjust to the new change and produce more. This will lead to hiring more labour units. Just for an illustration, let's analyse new isoprofit line

$$L = \left(\frac{p}{w}\right)^1 y - \frac{\Pi}{w} \tag{16}$$

and look for technologically feasible profit maximizing quantity of production. The analysis brings us to the point of tangency between the real variable cost curve and the isoprofit line. By changing real product price, another tangents are obtained and the cost curve is enveloped with tangents. At the real product price $\left(\frac{p}{w}\right)^0$ the optimal output level is y^0. The equation of a tangent on the real variable cost curve at $\left(\frac{p}{w}\right)^0$ is $L = \left(\frac{p}{w}\right)^0 y + \left(\frac{\pi}{w}\right)^0$ or equivalently, the equation of a line passing through the point $\left[y^0, L(y^0)\right]$ with the given slope $\left(\frac{p}{w}\right)^0$ is

$$L - L(y^0) = \left(\frac{p}{w}\right)^0 (y - y^0)\,. \tag{17}$$

At the real product price $\left(\frac{p}{w}\right)^1$ the optimal output level is y^1. The equation of a tangent on the production curve at $\left(\frac{p}{w}\right)^1$ is $L = \left(\frac{p}{w}\right)^1 y + \left(\frac{\pi}{w}\right)^1$ or equivalently, the equation of a line passing through the point $\left[y^1, L(y^1)\right]$ with the given slope $\left(\frac{p}{w}\right)^1$ is

$$L - L(y^1) = \left(\frac{p}{w}\right)^1 (y - y^1)\,. \tag{18}$$

Since the graph of the convex real variable cost function is above its tangent, the following inequalities for the two output levels hold: $L(y^1) \geq L(y^0) + \left(\frac{p}{w}\right)^0 (y^1 - y^0)$ and $L(y^0) \geq L(y^1) + \left(\frac{p}{w}\right)^1 (y^0 - y^1)$. From these two previous inequalities the important comparative statics result from the production theory follows,

$$(y^1 - y^0)\left[\left(\frac{p}{w}\right)^1 - \left(\frac{p}{w}\right)^0\right] \geq 0. \tag{19}$$

It implies that an increase in the real product price increases the product supply of the profit maximizing firm for the given technology and fixed input.

RECOVERING THE PRODUCTION FUNCTION AND THE REAL VARIABLE COST FUNCTION FROM THE NORMALIZED PROFIT FUNCTION

In the previous section we analysed the short run profit maximization model from two perspectives. From the first perspective technology in the short run was described by the short run production function and the real wage was given. The goal was to find the profit maximizing level of labour in the short run. From the second perspective the short run technology was described by the real variable cost function and the real product price, expressed in the units of labour, was given. The goal was to find the profit maximizing output level in the short run. In this section our goal is to recover the production function in description of the firm's technology by starting from the derived profit function. Also, to this kind of analysis we also add recovering the real variable cost function from the derived profit function.

In the primal problem, when the real wage was $\left(\frac{w}{p}\right)^0$ the profit maximizing producer chose L^0 and achieved maximum normalized profit expressed by the function $\pi\left(\frac{w^0}{p^0}\right)$. When the real wage changes, the producer can react in two ways. He can continue employing the same level of labour or can adjust to new market changes. In the first case we say that the producer is passive and his behaviour can be described geometrically with the line

$\frac{\pi}{p} = f(L^0, \overline{K}) - \frac{w}{p} L^0$ in the space where the real wage is on the horizontal axis and the profit is on the vertical axis. The intercept of the line on the vertical axis is $f(L^0, \overline{K}) = y^0$. In the second case we say that the producer is active and compared to the passive producer he will hire profit maximizing level of labour at every real wage and accomplish higher profit. Therefore, the graph of the maximum normalized profit function of the active producer is above the graph of the normalized profit function of the passive producer, which is the line and which is the tangent of the first graph. This is why the normalized profit function is convex in real wage. Furthermore, from the relationship between the profit of the active producer which is represtented by the function $\frac{\pi}{p}(\frac{w}{p}, \overline{K})$, and the profit of the passive producer, represented by the line $\frac{\pi}{p} = f(L^0, \overline{K}) - \frac{w}{p} L^0$, the following inequality follows

$$\frac{\pi}{p}(\frac{w}{p}, \overline{K}) \geq f(L^0, \overline{K}) - \frac{w}{p} L^o. \tag{20}$$

By changing places of the normalized profit function and the production function, we get

$$f(L^0, \overline{K}) \leq \frac{\pi}{p}(\frac{w}{p}, \overline{K}) + \frac{w}{p} L^o. \tag{21}$$

Since the level of labour L^0 is optimal at real wage $\frac{w^0}{p^0}$, the following holds

$$f(L^0, \overline{K}) = \frac{\pi}{p}\left[\left(\frac{w}{p}\right)^0, \overline{K}\right] + \left(\frac{w}{p}\right)^0 L^0. \tag{22}$$

Therefore, the short run production function is the result of minimizing the sum of the normalized profit and the real variable cost, where the choice variable is the dual variable, real wage,

$$f(L^0, \overline{K}) = \min_{\frac{w}{p}} \frac{\pi}{p}(\frac{w}{p}, \overline{K}) + \frac{w}{p} L^o. \tag{23}$$

By differentiating the goal function in (22) with respect to real wage, the first order necessary condition is obtained

$$L^0 = -\frac{\partial \frac{\pi}{p}(\frac{w}{p}, \overline{K})}{\partial \left(\frac{w}{p}\right)} \tag{24}$$

which has an important economic implication. It implies that the optimal labour quantity of the perfectly competitive, profit maximizing firm is obtained by simple differentiation of the normalized profit function with respect to the real wage and it is known in the literature as Hotelling's lemma.

It can be noticed that the solution to the primal optimization problem, in which the starting point was the short run production function and the normalized profit function was derived, is

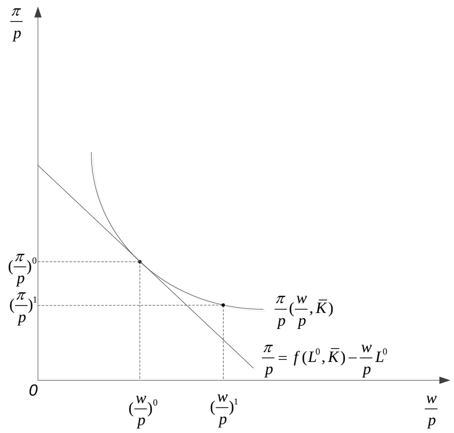

Figure 2. The profit function of the active and passive producer.

equal to the marginal profit, $L^0 = \dfrac{\partial \frac{\pi}{p}(\frac{w}{p}, \overline{K})}{\partial \left(\frac{w}{p}\right)}$, and that the solution to the dual optimization problem, in which the starting point was the normalized profit function and the short run production function is derived, is equal to the marginal product of labour, $\left(\dfrac{w}{p}\right)^0 = \dfrac{\partial f(L, \overline{K})}{\partial L}$.

Table 1. The symmetry of problem solving and the results of the models.

The primal problem	**The dual problem**
$\frac{\pi}{p}\left[\left(\frac{w}{p}\right)^0, \overline{K}\right] = \max\limits_{L} f(L, \overline{K}) - \left(\frac{w}{p}\right)^0 L$	$f(L^0, \overline{K}) = \min\limits_{\frac{w}{p}} \frac{\pi}{p}(\frac{w}{p}, \overline{K}) + \frac{w}{p} L^o$
First order necessary conditions:	**First order necessary conditions:**
$\left(\frac{w}{p}\right)^0 = \frac{\partial f(L, \overline{K})}{\partial L}$	$L^0 = -\frac{\partial \frac{\pi}{p}(\frac{w}{p}, \overline{K})}{\partial \left(\frac{w}{p}\right)}$

We conduct the same kind of analysis in recovering the short run variable cost function from the profit function. When the real product price was given, $\left(\dfrac{p}{w}\right)^0$, the optimal level of production was y^0. Producing this level of production the firm realized the maximum normalized profit, expressed in the labour units, equal to $\dfrac{\pi}{w}\left(\dfrac{p}{w}\right)^0$. If the real product price changes, the producer has two options. He can stay passive and produce the same output level or can adjust to new market changes. The behaviour of the passive producer can be described geometrically with the line $\pi = \dfrac{p}{w} y^0 - L(y^0)$ in the space where the real product price, expressed in the labour units, is on the horizontal axis and the normalized profit on the vertical axis. If the producer is active on the other hand, he will produce profit maximizing output level at every real product price and realize higher profit. By comparing the graph of the maximum normalized profit function of the active producer and the graph of the normalized profit function of the passive producer, which is the line, it can be concluded that the first graph is above its tangent, which means that the normalized profit function is convex in real product price. Since the maximum profit is always higher than the profit reached by producing the given output level, the following inequality holds

$$\frac{\pi}{w}(\frac{p}{w},\bar{K}) \geq \frac{p}{w}y^0 - L(y^0)\,. \tag{25}$$

By changing places of the normalized profit function and the real variable cost function, the following inequality follows:

$$L(y^0) \geq \frac{p}{w}y^0 - \frac{\pi}{w}(\frac{p}{w},\bar{K})\,. \tag{26}$$

For the output level y^0, which is optimal at real product price $\left(\frac{p}{w}\right)^0$, it follows

$$L(y^0) = \left(\frac{p}{w}\right)^0 y^0 - \frac{\pi}{w}\left[\left(\frac{p}{w}\right)^0, \bar{K}\right]. \tag{27}$$

Therefore, the real variable cost function is the result of the model of maximizing the difference between the real revenue and normalized profit, all expressed in the labour units, where the choice variable is the real product price,

$$L(y^0) = \min_{\frac{p}{w}}\left(\frac{p}{w}\right)y^0 - \frac{\pi}{w}\left[\left(\frac{p}{w}\right), \bar{K}\right]. \tag{28}$$

By differentiating the goal function with respect to the real product price the following result is obtained:

$$y^0 = \frac{\partial \frac{\pi}{w}(\frac{p}{w},\bar{K})}{\partial\left(\frac{p}{w}\right)}\,. \tag{29}$$

This result implies that the optimal output level of the perfectly competitive, profit maximizing firm is obtained by simple differentiation of the normalized profit function with respect to the real product price and it is known in the literature as Hotelling's lemma.

It can be noticed that the solution to the primal optimization problem, in which the starting point was the variable cost function and the normalized profit function was derived, is equal to the marginal profit, $y^0 = \frac{\partial \frac{\pi}{w}(\frac{p}{w},\bar{K})}{\partial\left(\frac{p}{w}\right)}$, and that the solution to the dual optimization problem, in which the starting point was the normalized profit function and the real variable cost function is derived, is equal to the real marginal cost, $\frac{p}{w} = \frac{dL(y)}{dy}$.

Table 2. The symmetry of problem solving and the results of the models.

The primal problem	The dual problem
$\frac{\pi}{w}\left[\left(\frac{p}{w}\right)^0, \bar{K}\right] = \max_y \left(\frac{p}{w}\right)^0 y - L(y)$	$L(y^0) = \min_{\frac{p}{w}} \left(\frac{p}{w}\right) y^0 - \frac{\pi}{w}\left[\left(\frac{p}{w}\right), \bar{K}\right]$
First order necessary conditions:	**First order necessary conditions:**
$\left(\frac{p}{w}\right)^0 = \frac{dL(y)}{dy}$	$y^0 = \frac{\partial \frac{\pi}{w}(\frac{p}{w}, \bar{K})}{\partial\left(\frac{p}{w}\right)}$

EXAMPLE

In illustration of duality between the short run profit, variable cost and production functions we start from the Cobb-Douglas production function $y = f(L,K) = L^{\frac{1}{3}} K^{\frac{1}{3}}$ in describing the firm's technology. Let's assume that the short run level of capital is 9 which brings us to the following short run production function $y = f(L,\overline{K}) = 3L^{\frac{1}{3}}$.

To derive the short run normalized profit function, the short run profit maximization problem has to be solved,

$$\frac{\pi}{p}(\frac{w}{p}, K = 9) = \max_L 3L^{\frac{1}{3}} - \frac{w}{p} L . \tag{30}$$

Since the choice variable in the above maximization model is the level of labour, by differentiating the goal function with respect to L, the first order necessary condition is obtained $\frac{w}{p} = L^{-\frac{2}{3}} / ^{-\frac{3}{2}}$, from which the standard result from microeconomic theory follows that the firm will hire that level of labour for which the marginal product is equal to real wage. By solving the equation, we get the labour demand function $L(\frac{w}{p},\overline{K}) = \left(\frac{w}{p}\right)^{-\frac{3}{2}}$. To obtain the supply function of a firm in the short run, we substitute the derived input demand function in the short run production function, $y(\frac{w}{p},\overline{K}) = 3(\frac{p}{w})^{\frac{1}{2}}$. Finally, substitution of the derived input demand function and the supply function in the goal function of the short run profit maximization model gives the short run normalized profit function, $\frac{\pi}{p}(\frac{w}{p},\overline{K}) = 3(\frac{w}{p})^{-\frac{1}{2}} - \frac{w}{p}\left(\frac{w}{p}\right)^{-\frac{3}{2}} = 2\left(\frac{w}{p}\right)^{-\frac{1}{2}}$.

An alternative approach in deriving the normalized profit function includes describing the technology with the cost function. The minimum amount of labour needed to produce a given output level is obtained by inverting the production function, $L(y) = \frac{y^3}{27}$. In this alternative primal problem the choice variable is the quantity of production and the short run profit maximization model is defined as

$$\pi(\frac{p}{w},\overline{K}) = \max_{y} \frac{p}{w} y - \frac{y^3}{27}. \tag{31}$$

By differentiating the goal function with respect to *y*, we get the standard result from the microeconomic theory that the perfectly competitive firm will supply the level of production for which the marginal cost is equal to the product price, $\frac{p}{w} = \frac{y^2}{9}$, $p = \frac{wy^2}{9}$. By solving the equation, the supply function is obtained, which is the same function as the one obtained in the first approach, $y(\frac{p}{w},\overline{K}) = 3(\frac{p}{w})^{\frac{1}{2}}$. By inserting the supply function in the goal function of the short run profit maximization model, the profit function is obtained, this time expressed in the units of labour, $\frac{\pi}{w}(\frac{p}{w},\overline{K}) = 3\frac{p}{w}\left(\frac{p}{w}\right)^{\frac{1}{2}} - \frac{p}{w}\left(\frac{p}{w}\right)^{\frac{1}{2}} = 2\left(\frac{p}{w}\right)^{\frac{3}{2}}$.

In the dual problem we start from the derived short run normalized profit function and our goal is to recover the short run production function. According to (12), the dual optimization problem is $f(L,\overline{K}) = \min_{\frac{w}{p}} 2\left(\frac{w}{p}\right)^{-\frac{1}{2}} + \frac{w}{p}L$. By differentiating the goal function with respect to the real wage, the first order necessary condition is obtained, $-\left(\frac{w}{p}\right)^{-\frac{3}{2}} + L = 0 \Rightarrow \frac{w}{p} = L^{-\frac{2}{3}}$. By inserting the optimal real wage function in the goal function of the dual problem, the production function is obtained $f(L,\overline{K}) = 2(L^{-\frac{2}{3}})^{-\frac{1}{2}} + L^{-\frac{2}{3}}L = 3L^{\frac{1}{3}}$.

Alternatively, we can start from the derived normalized short run profit function and recover the cost function. Following this path, the dual optimization problem is $L(y) = \max_{\frac{p}{w}} \frac{p}{w}y - 2\left(\frac{p}{w}\right)^{\frac{3}{2}}$. By differentiating the goal function with respect to the product price expressed in the units of labour, $\frac{p}{w}$, the first order necessary condition is obtained, $y = 3\left(\frac{p}{w}\right)^{\frac{1}{2}}$. Once more, the Hotelling's lemma is confirmed, by which the supply function of a perfectly competitive firm is obtained by differentiating the profit function with respect to $\frac{p}{w}$. The optimal product price expressed in the units of labour is equal to $\frac{p}{w} = \frac{y^2}{9}$ and the variable cost function expressed in the units of labour is recovered, $\frac{y^3}{9} - \frac{2y^3}{27} = \frac{y^3}{27}$.

CONCLUSION

In this article the relationship between the short run production function, variable cost function and the normalized profit function in a convex analysis framework is analysed. The goal was to deductively apply the results of convex analysis on the known short run profit maximization problem. In the primal optimization model the technology in the short run is represented by the short run production function and the normalized profit function, which expresses profit in the output units, is derived. In this approach the choice variable is the labour quantity. Alternatively, technology is represented by the real variable cost function, where costs are expressed in the labour units, and the normalized profit function is derived, this time expressing profit in the labour units. The choice variable in this approach is the quantity of production. The emphasis in these two perspectives of the primal approach is given to the first order necessary conditions of both models which are the consequence of enveloping the closed convex set describing technology with its tangents. The dual model includes starting from the normalized profit function and recovering the production function, and alternatively the real variable cost function. In the first perspective of the dual approach the choice variable is the real wage, and in the second it is the real product price expressed in the labour units. It is shown that the change of variables into parameters and parameters into variables leads to both optimization models which give the same system of labour demand and product supply functions and their inverses. By deductively applying the results of convex analysis the comparative statics results are derived describing the firm's behaviour in the short run.

Due to the basic behavioural assumption in economics about the optimization subject to constraints of economics agents and the important role of convex sets in characterizing economics laws, microeconomic phenomena have a unique structure, which was in this article analysed in a simple and intuitive way and can be applid to any microeconomic problem which can mathematically be represented as an optimization model. It was assumed that the starting production function is differentiable but the results can be generalized to the nondifferentiable case what we leave for the future research.

Although the advantages of duality results are especially important from an empirical standpoint, one needs to be careful because the research reveals differences in estimates obtained by the primal and the dual function.

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