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SOME REMARKS ON THE EFFECTS OF PRODUCTIVITY ON GROWTH

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SUMMARY

Standard economic models predict that a *ceteris paribus* increase in the overall productivity results in an increased production if the economy departs from an equilibrium state. We show that this result is valid under specific conditions In other cases, even if the initial conditions of an economic system are so that the economy converges into (or starts from) an equilibrium state, the increase of overall productivity generally results, in the long run, in the collapse of the economy due to the unbalanced change in the money holdings. Hence, in general, an appropriate expansive monetary policy should accompany the increase of productivity.

KEY WORDS

economic growth and aggregate productivity, multisector growth, monetary growth

CLASSIFICATION

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INTRODUCTION

Our purpose here is to examine the conditions under which the standard economics' result on the effect of a change in overall productivity on production holds. This standard result is intuitive: a ceteris paribus increase in overall productivity increases production. (see e.g. [1]).

To be able to predict the effect of any shocks or economic policy, one needs an appropriate representation of our economies.

Standard economic analysis [2] makes this representation by dividing time into periods defined by the actions that one can execute in the period. These actions are production, consumption and exchange. The notion of short or long run is used in several senses. It can refer either to the number of exogenous variables during the optimization problem of economic agents, or (in a sequential model) to the number of periods during which a given (equilibrium) state is reached.

A representation of the complex system of our economies naturally requires shortcuts and simplifications. Economic theory is intended to explain the choice of key variables and connections (causalities) between them. As a result, in standard economic models, the general hypotheses/results are: the neutrality of money in production as well as in consumption [3], and the positive effect of the increase of overall productivity on production.

Ayres and Martinás [4 – 8] showed that a re-foundation of the assumptions on the behaviour of economic agents questions the habitual results; the teachings of standard economic analysis being just a special case. Moreover, this reformulation permits to handle other theoretical findings, which are non-conform to the standard formulation[1]. As in Ayres and Martinás' work, agents do not seek the traditional profit and utility maximum, the neutrality of money does not necessarily hold any more in production; as a result an increase in productivity changes the commodity/money ratio and usually the economy is driven away from a stable equilibrium state and collapses after a while. It follows that an appropriate monetary policy accompanying changes in production is needed to save the standard results.

The paper is organised as follows. First, we consider a simplified version of the Ayres-Martinás model, next we examine the conditions under which the Ayres-Martinás model has the same properties as the standard economic models, and finally we discuss the effects of productivity change on production in the Ayres-Martinás model.

A SIMPLIFIED AYRES-MARTINÁS MODEL

Let us consider a simplified model of Ayres-Martinás (for a detailed discussion see [5]).

We construct two macro-agents, called sector 1 and sector 2. A sector comprises not only firms but also consumers drawing their revenues from the sector. We suppose that sectors are disjunctive. We omit banks as usual in standard models and consider money exogenously[2].

Time is divided into periods, which in turn are divided into sub-periods. In the first sub-periods sectors produce, in the second sub-period they exchange and in the third they consume. Hence, the stocks x of a sector changes as follows:

$$\text{sector 1: } x^1_t = x^1_{t-1} + x^1_{p,t} + x^1_{e,t} - C^1_t,$$

$$\text{sector 2: } x^2_t = x^2_{t-1} + x^2_{p,t} + x^2_{e,t} - C^2_t,$$

where x^2_t is the stock x held by sector 2 at the end of period t, $x^2_{p,t}$ is the change of stock (flow) x of sector 2 in the period t due to the production process.

We suppose that there are four stocks: commodity y_1 produced exclusively by sector 1, commodity y_2, produced exclusively by sector 2, labour L and money M. There is no depreciation. For the sake of simplicity, we consider a linear production technology, i.e.: $x_p = a_l$, where $a = (a_1, a_2, a_L)$ is the input-output vector; by assumption for sector 1 $a_1 > 0$, $a_2 \le 0$ and $a_L < 0$; for sector 2 $a_2 > 0$, $a_1 \le 0$ and $a_L < 0$; l is the level of production.

Sectors, being economic agents, seek to get richer for economic decisions, hence we suppose that

- they can evaluate wealth. This evaluation is given by the following wealth function, identical for both sectors,

$$Z(y_1, y_2, L, M),$$

- they avoid avoidable losses for economic decisions in time:

$$\mathrm{d}Z/\mathrm{d}t \ge 0.$$

For production decisions from 2, if stocks are not binding, with linear approximation we obtain:

$$x_p = a_l = a \cdot K \cdot v^T \cdot a,$$

where v^T is the transposed row vector of values $v^T = (y_1/M, y_2/M, L/M)$, K is a parameter showing the intensity of reaction of the sector, i.e. a unit value added induces K times increase in the level of production.

For exchange decisions from 2, if stocks are not binding, with linear approximation we obtain:

$$x_e = N(v - p)$$

where N is a parameter showing the intensity of reaction of the sector in response to the surplus realisable on a commodity and p is the price offered.

For the sake of comparability with the standard model, let us suppose that exchange takes place always at equilibrium prices. We suppose that $N_1 = N_2 = N$. in that case, $p = (v_1 + v_2)/2$. By assumption, there is no exchange of labour. Consumption is given exogenously: c % of the commodities is consumed. The quantity of labour is fixed. We measure overall production as usual by the real GDP: first period prices multiplied by the quantities produced.

THE STANDARD ECONOMIC MODEL AS SPECIAL CASE OF THE AYRES-MARTINÁS MODEL

Standard economic models are equilibrium models. This means that:

- exchange takes place always at equilibrium prices, i.e. prices at which planned demand (plans being made before exchanges) in a period equals planned supply. Hence, the reallocation of stocks by exchange is independent from the exchange mechanism. In other terms, there are no exchange mechanisms but just price adjustment mechanisms in these models [9]. As a result, money is neutral in exchange,
- (planned) excess demand is always equal to zero in the current period referred often to as short run equilibrium. In multi-period models agents make expectations on future prices. The situation where planned excess demand is zero in all periods is called long run equilibrium. The suite of short run equilibriums is called stationary equilibrium.

One can easily check that in general in the Ayres-Martinás dynamic model there is no equilibrium as in standard models even if (as we have also chosen the specification of the Ayres-Martinás model)

1. *the evaluation function* (utility function in the standard model and the wealth function in the Ayres-Martinás model) *is well behaved* (strictly quasi-concave, strictly increasing twice differentiable function); and the production technology inhibits the Arrow-Debreu

[10, 11] model's assumptions (production set determined by the technology is nonempty, closed, convex, irreversible, possibility of inactivity, no output without using some inputs),

2. *sectors are symmetrical*, that is to say:
   - they have the same production technology $a_1^1 = a_2^2$, $a_2^1 = a_1^2$ and $a_L^1 = a_L^2$,
   - they have the same reaction parameters K and N,
   - they consume the same proportion c,
   - they have the same stocks in symmetry i.e.: $y_1^1 = y_2^2$, $y_2^1 = y_1^2$, L and M are the same,
3. *exchange takes place only at equilibrium prices* i.e. while prices evolve in a sub-period stocks do not change.

A typical path of the economy with the above assumptions is as shown in Fig. 1.

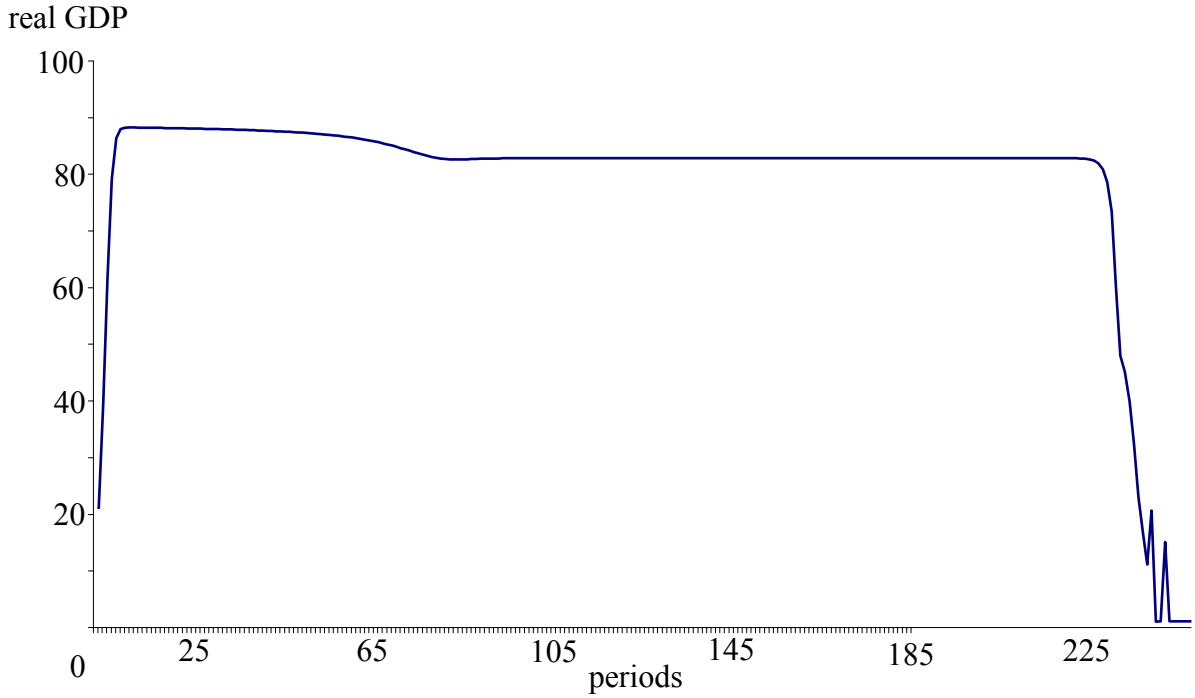

Fig. 1. Typical economic path.

That is to say, after a while, economy collapses because of the non-appropriate (non proportional) change in stocks. An appropriate change (in our case increase) of money supply permits to avoid the collapse of the economy. As an illustration, we show in the figure below an appropriate (i.e.: permits production in the long run) and an excessive increase in the money stocks, Fig. 2.

That figure points that the elimination of money balance effects in exchange by the assumption of exchange at equilibrium prices is not sufficient to assure, in general, stationary equilibrium even with the above enumerated additional simplifications. Neutrality of money (i.e.: no effect on exchange and/or production decisions) should be assured also for production decisions to obtain the standard economic findings.

This is the case of standard profit maximising behaviour. Formulated in the Ayres-Martinás framework for sector 1:

$$\max_{y_1^v y_2^v, L^v} y_1^v, y_2^v L^v M$$

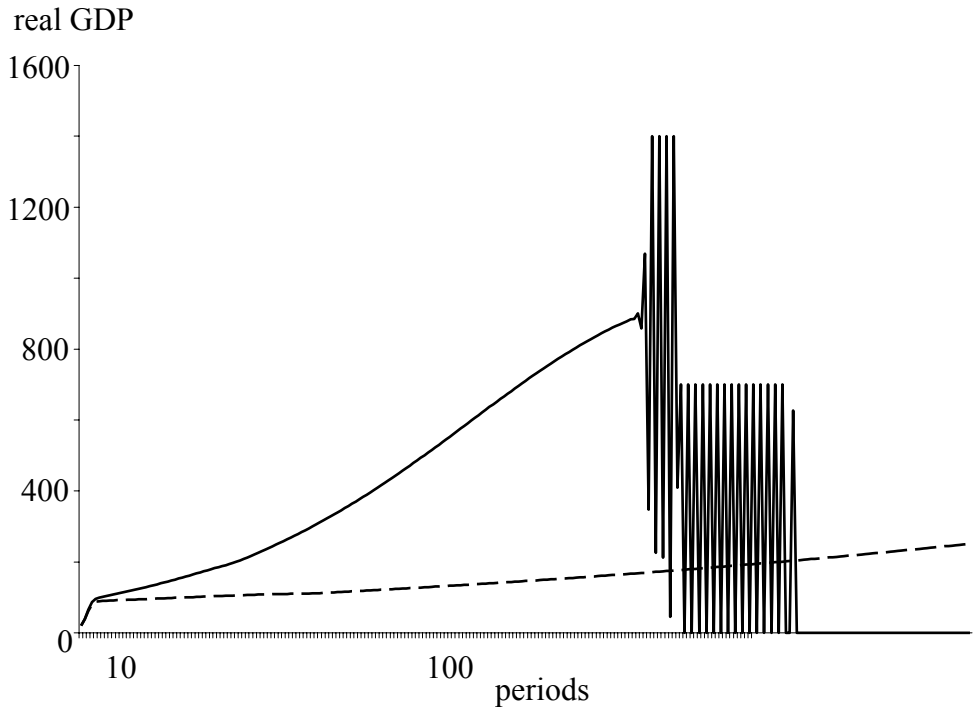

Fig. 2. Expansive monetary policy. Solid (dashed) line denotes high (low) expansion of M.

$$s.t.: \begin{pmatrix} y_1 \\ y_2 \\ L \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_L \end{pmatrix} l$$

$$y^{\mathrm{v}}_{\mathrm{j}} = y^{\mathrm{i}}_{\mathrm{j}} + y\mathrm{j}, \quad j = 1, 2,$$
$$L^{\mathrm{v}} = L_{\mathrm{i}} + L.$$

where $y^{\mathrm{v}}_{\mathrm{j}}$ is the stock of sector 1 at the end of the production period in product j, $y^{\mathrm{i}}_{\mathrm{j}}$ is the endowment of sector 1 at the beginning of the production sub-period and y_{j} is the change of stock j during the production process (l is the level of production already defined).

Without loss of generality we suppose that $a_1 = 1$. Hence, we can transcribe the above problem in the following manner:

$$\max_{y_1^v\, y_2^v, L^v} y_1^v, y_2^v L^v M$$

$$s.t.: y_j = \min\left\{\frac{y_2}{a_2}; \frac{L}{a_L}\right\}$$

$$y^{\mathrm{v}}_{\mathrm{j}} = y^{\mathrm{i}}_{j} + y_{\mathrm{j}}, \quad j = 1, 2,$$
$$L^{\mathrm{v}} = L^{\mathrm{i}} + L.$$

It is straightforward that the in optimum production does not depend on the quantity of money. In that case, the standard result on the existence of stationary equilibrium holds.

PRODUCTIVITY AND GROWTH: SOME RESULTS

Standard economic model predicts that a ceteris paribus increase in the overall productivity increases production in the short and in the long run. As we have seen this result is true in every case when stocks change proportionally (or do not change). This can be assured for all cases either

- by assuming that consumption is always so that leaves stocks at the level of the beginning of the period; or
- by assuming that money is neutral. This is assured for example if we assume that firms maximise profits for one period (given their expectations on future).

If we omit the above hypotheses, we cannot assure, in general, the existence of stationary equilibrium and hence production collapses after a time. That is to say, in the short run it is always true that an increase in overall productivity increases production. But in the long run production can decrease because of disproportional variations.

However, if the parameters (production response intensity K, exchange response intensity N, technology a, initial stocks) of the considered model are so that there exists stationary equilibrium we can say a little bit more. In that case, still referring to the symmetric case, a "small" increase in productivity gives the traditional result. The stationary equilibrium, depending on the concrete values of the parameters, is more or less sensitive to these parameters. However, in any case when the symmetric behaviour of the sectors is broken (K, N is different) the initial stationary equilibrium brakes down with an overall productivity increase resulting in the collapse of production.

A straightforward solution to avoid the collapse of production is to consider not a *ceteris paribus* change in the overall productivity, but a simultaneous change in the money supply. For, in the Ayres-Martinás model, production depends on the evaluation of agents, which in turn depends on the money holdings. Hence, the variation of money holdings also effects production.

CONCLUSIONS

Following the proposition of standard economics a ceteris paribus increase in overall productivity increases production. We have examined this proposition in the light of Ayres-Martinás works. These authors have re-founded the usual behavioural assumptions on economic agents; their theory incorporating as a special case the standard economic theory.

We have found that this standard proposition holds for the short run, but for the long run, in general, without an appropriate monetary policy accompanying the productivity increase, production decreases in the long run even if the economy departs from a stationary equilibrium state. The reason for this is the following: all economic decisions depend on how agents evaluate their stocks. As money holdings play in that evaluation and, in general, non-proportional changes occur, some agents loose all their money holdings and cannot take part any more in the production process.

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REMARKS

[1]For example, balance sheet approaches [6, 12 – 14].

[2]Opposed to the balance sheet approach following which this cannot be done.

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NAPOMENE O UČINKU PRODUKTIVNOSTI NA RAST

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SAŽETAK

Standardni ekonomski modeli predviđaju kako, uz sve ostale uvjete nepromijenjene, porast opće produktivnosti rezultira povećanom proizvodnjom ako se ekonomski sustav udaljuje iz ravnotežnog stanja. U radu je pokazano kako taj rezultat vrijedi uz određene uvjete. U ostalim slučajevima, čak i kad su početni uvjeti ekonomskog sustava takvi da ekonomija teži (ili započinje u) ravnotežnom stanju, porast opće produktivnosti dugoročno rezultira kolapsom ekonomije zbog neuravnotežene promjene u čuvanju novca. Zbog toga u općem slučaju porast produktivnosti mora biti praćen približno ekspanzivnom monetarnom politikom.

KLJUČNE RIJEČI

ekonomski rast i agregirana produktivnost, višesektorski rast, monetarni rast