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MATHEMATICAL MODELS AND EQUILIBRIUM IN IRREVERSIBLE MICROECONOMICS

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ABSTRACT

A set of equilibrium states in a system consisting of economic agents, economic reservoirs, and firms is considered. Methods of irreversible microeconomics are used. We show that direct sale/purchase leads to an equilibrium state which depends upon the coefficients of supply/demand functions. To reach the unique equilibrium state it is necessary to add either monetary exchange or an intermediate firm.

KEY WORDS

irreversible microeconomics, mathematical models, thermodynamics

CLASSIFICATION

JEL: C63, D01, D83

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MODELS OF IRREVERSIBLE MICROECONOMICS

Microeconomics includes the subject of the interaction between economic agents. Each agent is a set of individuals. Number of individuals is huge in real economic systems and it is not possible to control each of them. Averaged parameters can be used as controls in these systems. Prices are parameters of that kind. The same situation is in thermodynamic systems where parameters averaged on the set of particles can be controls. These parameters in thermodynamics are temperatures, concentrations etc. That is why microeconomic systems as thermodynamic ones can be considered as macrosystems [1]. In this sense they are analogous to thermodynamic systems. A state of the economic agent is described by stocks of resources. One of the resources should act as money: this resource is called a *basic resource*. Each economic agent needs money.

The system is *isolated* if there is no resource to exchange between the system and its surroundings. The system is closed if the system and its surroundings can exchange only one resource and this resource is basic one. An economic system is open if there is more than one resource to exchange with the system's surroundings.

We mention some facts which should be accounted for in a mathematical model:

- there are several processes in economic systems. They are the exchange of resources, consumption, and production. We assume that behaviour of economic agents is rational. It means that states of the agents are comparable. Thus, we can introduce a *wealth function* depending on the economic agent state [2] and postulate that the wealth function is an objective function for an economic agent.
- the lower the price the lower the intensity of sales of economic agents. There exists a price when the intensity is equal to zero. This price is called the agent's value of the resource. It means that if the price is greater than the value then the agent sells the resource, and if the price is greater than the value then the agent buys the resource. If the price is equal to the value then there is no exchange process; the agent is in equilibrium state with its environment.
- resource flows between economic agents in an isolated system without consumption and production processes run down in time and tend to zero. In the extreme case all economic agents appraise resources equally.
- it is impossible to increase the wealth of an economic agent due to resources exchange with another agent so that the stocks of the latter at the beginning and the end of the process are the same.
- extraction of money is possible in a closed system with the help of an intermediate firm only. In this process the firm should buy and sell the resources at different prices.

These facts need an adequate mathematical description. Some features are familiar to thermodynamic systems, others are different. There are many published works devoted to thermodynamic approaches to economic processes [3 - 8], but irreversible microeconomics is at a nascent stage. Many problems are yet unsolved, e.g., the determination of the optimal regime of the intermediate firm under different conditions.

Two features of economic systems are important. One of them is called the voluntary principle. It means that it is not possible to donate any amount of a resource. Each transfer of a resource should be indemnified by another resource.

The other feature is the following. Spontaneous processes of interaction exist in any economic macrosystem. These processes do not demand any changes in the neighbourhood. However, some external influence is necessary to return the system to its initial state. These processes are irreversible. We need a value characterizing the degree of irreversibility of such systems. The value reaches its maximum at the equilibrium state like entropy in thermodynamics.

Let us denote by $N = (N_1; \ldots, N_k)$ the vector or resources stocks and by M the amount of the basic resource. For all economic agents M has a common dimension. In other words, all agents use the same currency as money. Values N and M are extensive values: if we divide a homogenous economic agent the values divide in the same proportion. Intensive values exist as well. They are the value of resources $p = (p_1, \ldots, p_k)$. The values depend on resources stocks. A common assumption is that if several economic agents are in equilibrium state then the values do not change. So, values are assumed to be homogeneous functions of zero degree.

Value of the resource p_i is the minimal (maximal) price in units of the basic resource when the economic agent is ready to sell (buy) the resource with the infinitesimal small intensity.

In exchange processes economic agents can play the role of other seller or buyer. They are characterized by demand functions. The demand function for resource i shows the intensity of purchasing the resource with respect to its value p_i and price c_i. Demand function $n_i(p_i, c_i)$:

$$\mathrm{sign}[n_i(p_i, c_i)] = \mathrm{sign}(p_i - c_i), \tag{1a}$$

$$n_i(p_i, c_i) = 0 \quad \text{if } c_i = p_i, \tag{1b}$$

$$\frac{\partial n_i}{\partial c_i} < 0 \quad \& \quad \frac{\partial n_i}{\partial p_i} > 0, \quad i = 1, \ldots k, \tag{1c}$$

determines the kinetics of resource exchange processes. The value of n_i can either be positive when the agent buys the resource or negative when the agent sells the resource. Recall that the dimension of both price and value is the ratio of the basic resource to the resource units.

There are three kinds of economic agents:

- the values of some economic agents depend on the stocks of resources. Let us call such agents *passive economic agents* or *economic agents with finite capacity*. The larger the stock of a resource the less its value. The larger the stock of the base resource the larger the value of all other resources.
- then there are agents whose values do not depend upon the stocks of resources. Let us call them *economic agents with infinite capacity*, *economic reservoirs* or *markets*. Note that the introduced terms do not purport absence of free will of individuals. It was postulated that the agent can consist of a large number of individuals. Due to the law of large numbers averaged behavior of the individuals allows one to introduce regular laws like supply/demand functions. These laws can either depend on amount of accumulated stocks of the resources or not. It is occasioned by scale of the agent. In the case of enormous scale of the agent resource fluxes cannot change the stocks significantly, that is why values of such an agent are either constant or changed due to exogenic factors.
- Intermediate firms are economic agents who can change their values voluntarily. These agents either set prices or intensities of resources flows so as to maximize the intensity of extracted basic resources. This firm works like the working fluid of a heat engine. It can establish contact with other economic agents and its relationships are characterised by controls.

WEALTH FUNCTION AND ITS CHARACTERISTICS

Macrosystem approach to economic agents allows us to introduce a function S depending on N and M. Each agent wants to maximize this function depending on its economic agent type. Following [2] we shall call it a *wealth function*. In early works [3, 5] this function was called a structure function. The wealth function and the values of resources are interrelated by the following ratios:

$$p_0 = \frac{\partial S}{\partial M}, \quad p_i = \frac{1}{p_0}\frac{\partial S}{\partial N_i}, \quad \text{i = 1, … k,} \tag{2}$$

The wealth function is homogenous of degree one. It means that if we multiply all arguments by a constant then the result will be multiplied by the same constant. Therefore, we can rewrite it using the Euler theorem:

$$S(N,M) = p_0\left(M + \sum_i p_i M_i\right) = p_0 U(N,M), \quad p_0(N,M) > 0\,. \tag{3}$$

The proof of the existence of the wealth function was provided by L.I. Rozonoer and is published in the Appendix of review [7]. In [9] the proof was extended to the general case of *k* resources. The term *capitalization* (the *U* function) was introduced as an internal estimation of the cost of all resources collected by an economic agent. The principle of voluntary exchange does not permit any exchange process if the wealth of at least one economic agent decreases during the process.

In order to prove the existence of entropy in thermodynamic systems we need to introduce an integrating multiplier which has the same dimension K^{-1} for any subsystem. The dimension of the integrating multiplier p_0 introduced for the economic system depends on the dimension of the wealth function. The dimension of the wealth function can be different for different agents because during the exchange process the value of money will not be equal. We will show that this feature leads to the dependency of the equilibrium state on the kinetics of exchange (the *demand function*).

One interesting case of economic system occurs when there are several currencies there. Equilibrium state in an isolated system with foreign exchange market corresponds to equal values of currencies. If the market is an economic reservoir then p0 for each economic agent determines by equilibrium price of corresponding currency at the market. It means that in this case value p_0 does not depend on state of the economic agent.

In the following sections we will consider equilibrium states in isolated and closed systems and in systems including intermediate firms.

DIRECT EXCHANGE OF RESOURCES

In an isolated system the total amount of resources does not change as there is neither consumption nor production. Let us assume that

- the amount of money as a proportion of the amount of other resources is constant. In other words, the sum of intensities of money fluxes equals zero,
- during the voluntary exchange of resources economic agents with relatively high values p_{ij} (*i* denoting a resource and *j* an economic agent) buy the resource and those with relatively low values sell the resource.

In that case, we can introduce value of cost of resources $F = \Sigma_i\, p i N_i$. During exchange process wealth function does not decrease. Rate of *F* changing does not decrease, too. So, we have the following expression:

$$\sigma \equiv \frac{dF}{dt} = \sum_j \sum_i p_{ij}(N_j, M_j)\frac{dN_{ij}}{dt} \geq 0,\,. \tag{4}$$

where σ can be interpreted as the sum of the rates of capitalizations of economic agents when the values of money p_{0j} are constants.

Equilibrium state is stable. In this state value of resources cost is maximal and resources fluxes $n_{ij} = dN_{ij}/dt_i$ are equal to zero.

Let us consider a system where the exchange of resources takes place. If resources fluxes are not infinitely large the equilibrium state is reached asymptotically. However, the equilibrium is not unique. It is a function of the parameters of the exchange kinetics.

In the equilibrium state the following balance equations are true:

$$\sum_{j=1}^{n} \overline{N_{ij}} = N_{i0}, \quad \text{i = 1, … k,} \tag{5}$$

$$\sum_{j=1}^{n} \overline{M_j} = M_0. \tag{6}$$

Here $\overline{M_{ij}}$ and $\overline{N_{ij}}$ are equilibrium values of money and resources stocks. Adding equilibrium conditions to equations (5) and (6):

$$p_{ij}(\overline{N_j}, \overline{M_j}) = \overline{p}_i, \quad \text{i = 1, … k;} \quad \text{j = 1, … n.} \tag{7}$$

There are $(k + 1)n + k$ unknowns in the system of equations whereas the number of equations (5-7) is $k(n + 1) + 1$. Taking into account that $n > 2$ we have more unknowns than equations. That is why the equilibrium state is determined not uniquely. The equilibrium state depends on the kinetics of exchange.

A consequence of the characteristics of functions (1) is that the capitalization of each economic agent cannot decrease. So, the following inequalities are true.

$$\sum_{i=1}^{k} (\overline{p}_{ij}\overline{N_{ij}} - p_{ij0}N_{ij0}) + \overline{M_j} - M_{j0} \geq 0, \quad \text{j = 1, … n.} \tag{8}$$

We can solve for the equilibrium state if we add equations for the kinetics of resources exchange between the j-th and ν-th agent:

$$n_{ij\nu}(p_{ij}, c_{ij\nu}) = -n_{ij\nu}(c_{ij\nu}, p_{ij}), \tag{9}$$

$$m_{j\nu} = \sum_{i=1}^{k} c_{ij\nu} n_{ij\nu}(p_{ij}, c_{ij\nu}) = -m_{\nu j} \quad \text{j, ν = 1, … n.} \tag{10}$$

These equations allow us to derive the dependence of price on time. The dependency of the system state on time can be found from the differential equations,

$$\begin{aligned} \frac{dM}{dt} &= -\sum_{i=1}^{k}\sum_{\nu=1}^{n} c_{ij\nu} n_{ij\nu}(p_{ij}, c_{ij\nu}), \\ \frac{dN_{ij}}{dt} &= \sum_{\nu=1}^{n} n_{ij\nu}(p_{ij}, c_{ij\nu}), \quad j = 1,...n; \ i = 1,...k, \end{aligned} \tag{11}$$

with given inintial states. The solution of these equations tends to the equilibrium state as $t \to \infty$.

As an example, let us consider the case of a scalar resource exchange between two agents. At each moment of time, the price of the resource is determined by the equation

$$n_1(p_1, c) = n_2(p_2, c), \tag{12}$$

This price should fulfill the inequality

$$p_1 \geq c \geq p_2, \tag{13}$$

to meet the principle of voluntary exchange. So, if the equations of resource exchange kinetics are linear

$$n_1(p_1, c) = \alpha(p_1 - c), \tag{14}$$

$$n_2(p_2, c) = \alpha(p_2 - c), \tag{15}$$

then the price can be found from the equation

$$c(p_1, p_2, \alpha_1, \alpha_2) = \frac{\alpha_1 p_1 + \alpha_2 p_2}{\alpha_1 + \alpha_2}. \tag{16}$$

Furthermore

$$\frac{dM_1}{dN_1} = \frac{dM_2}{dN_2} = -c, \quad dN_2 = -dN_1, \tag{17}$$

$$N_1(0) = N_1^0, \qquad M_1(0) = M_1^0,$$

$$N_2(0) = N_2^0 = N_0 - N_1^0, \quad M_2(0) = M_2^0 = M_0 - M_1^0.$$

These conditions allow us to express c, M_1, M_2, N_2 as functions of N_1. The change in capitalization can be calculated as

$$\Delta U_1 = \int_0^{\overline{N_1}} [p_1(N) - c(N)]dN, \tag{18}$$

$$\Delta U_2 = \int_0^{\overline{N_1}} [c(N) - p_2(N)]dN, \tag{19}$$

for each agent and

$$\Delta U = \Delta U_1 + \Delta U_2 = \Delta F = \int_0^{\overline{N_1}} [p_1(N) - p_2(N)]dN, \tag{20}$$

for the system as a whole. In equations (18 - 20) values p_1, p_2 depend on N_1 only, because other variables are functions of N_1. In an isolated system, the total amount of money does not change, that is, $\Delta U = \Delta F$.

Let us use the Edgeworth diagram to illustrate the model. This diagram consists of two sets of equipotential lines of the wealth functions for both agents. One set of the lines is rotated through 180^o, initial point of the system of axes is at point (M_1^0, N_1^0).

Note that differential

$$dS = \frac{\partial S}{\partial N} dN + \frac{\partial S}{\partial M} dM = 0,$$

holds along an equipotential line.

According to (3) a slope of a tangent to an equipotential line equals to $\partial M = \partial N = -p$. It means that all points where two equipotential lines touch each other meet equilibrium conditions (7). A set of these points is the equilibrium set (contract line). However, it is not impossible to reach any point of the contract line from the given initial state during the finite period of time.

A point on the contract line is accessible if conditions (13) are satisfied. These conditions guarantee that the wealth function and capitalization cannot decrease for both agents. It is not possible to move along the contract curve without decreasing the wealth of one agent. So the set of equilibrium states is a Pareto set.

The equilibrium state depends on the equations of resource exchange kinetics. The state cannot be reached in finite time. Sets of attainability for linear laws of kinetics and trajectories of exchange in the Edgeworth diagram are shown in Fig. 1.

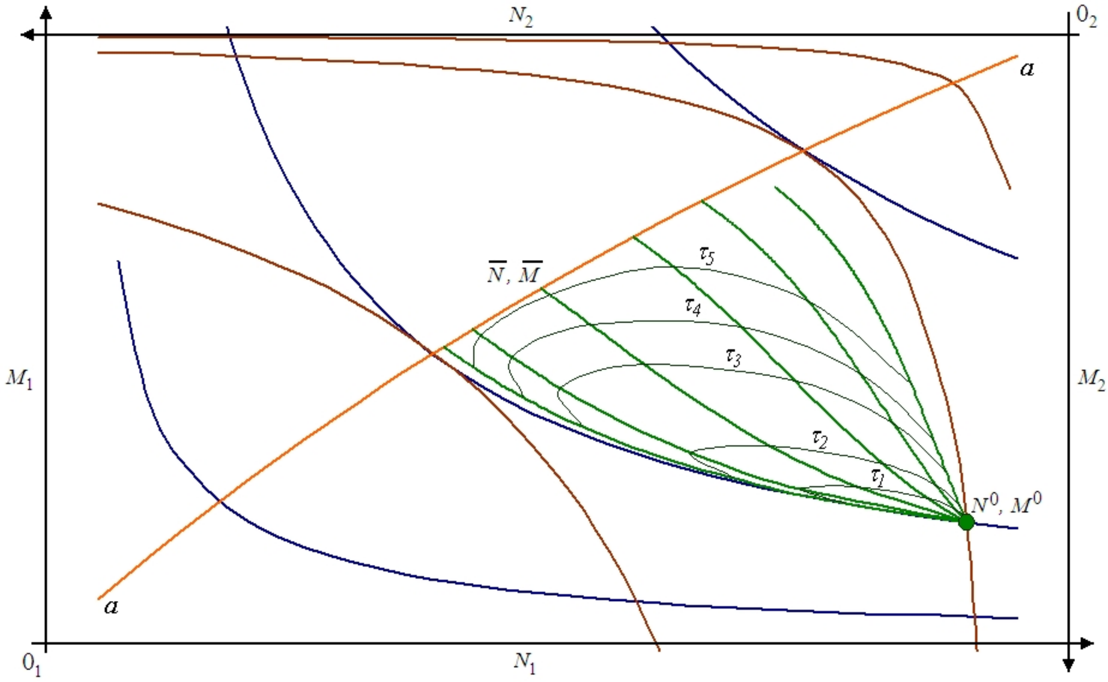

Figure 1. Edgeworth diagram of sets of attainability for linear laws of kinetics and trajectories of exchange.

PROFITLESS AUCTION SALE

Let price c be fixed at the value $c = p$. Then the trajectory of the system state is a straight line. The slope of the line is c. It should be chosen to equal the values of the resources at the equilibriums state. In this case the capitalization of both agents will be constant. The corresponding economic process is a process of profitless auction sale.

The equilibrium conditions consist of the balance equations (5) and equations (7). Prices c should be fixed on the right-hand side. The following equations are true for money stocks:

$$\overline{M_j} = M_j(0) - \sum_{i=1}^{k} \bar{c}[\overline{N_{ij}} - N_{ij}(0)], \quad \text{j = 1, …, n.} \tag{21}$$

From these equations we obtain (6) after taking (5) into account.

In sum, we have $k(1 + n) + n$ equations in as many unknowns. That is why the equilibrium state is determined. An illustration of profitless auction sale is the market for electrical energy where power stations and consumers propose their supply and demand dependencies respectively and a dispatcher assigns the equilibrium price to equalize supply and demand. The equilibrium state in the resource exchange at a profitless exchange corresponds to the maximum of the sum of wealth functions. The problem is to maximize the sum of two wealth functions subject to the resources constraint conservation laws. The Lagrangian is

$$L = S_1(\overline{N_1}, \overline{M_1}) + S_2(\overline{N_2}.\overline{M_2}) + \lambda_1(\overline{N_1} + \overline{N_2}) + \lambda_2(\overline{M_1} + \overline{M_2}).$$

The stationary conditions are

$$\frac{\partial S_j}{\partial \overline{N_j}} = \lambda_1, \quad \frac{\partial S_j}{\partial \overline{M_j}} = \lambda_2, \quad \text{j = 1, 2.} \tag{22}$$

Because

$$\frac{\partial S_j}{\partial \overline{M_j}} = p_{j0}(\overline{N_j},\overline{M_j}), \quad \frac{\partial S_j}{\partial \overline{N_j}} = p_{j0}(\overline{N_j},\overline{M_j})\,p_j(\overline{N_j},\overline{M_j})$$

both the values of the resources and the values of money should be equal for all agents. The result holds if both wealth functions have the same dimension.

Price *c* is equal to the value p in the equilibrium state. This value is used in equation (7). Conditions (5 - 7) should be supplemented with the following equation:

$$\overline{M_2} = (N_0^2 - \overline{N_2})\overline{p} + M_2^0. \tag{23}$$

It allows us to determine the final state (Fig. 2).

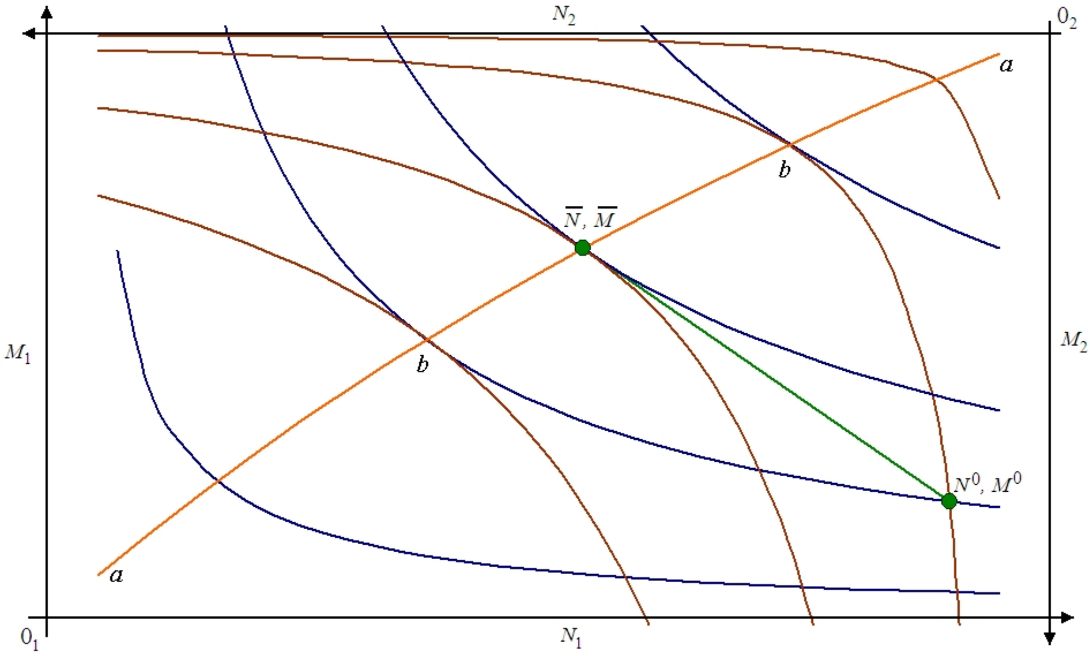

Figure 2. Edgeworth diagram of final state in the example of two-state profitless auction sale.

EXCHANGE BY BARTER

We have a system of agents. Each agent has stocks of several resources. The initial amounts and their values are known. During the exchange process the quantity of money of each economic agent is fixed. The kinetics of resources exchange is given by

$$\frac{dN_j}{dt} = \sum_{\nu=1}^{n} A_{j\nu}\Delta p_{j\nu}, \quad \nu \neq j, \quad j = 1, \ldots, n, \tag{24}$$

where $A_{j\nu}$ are the kinetic coefficients, $\Delta p_{j\nu}$ are differences in the values of the resources. In equilibrium $\Delta p_{j\nu} = 0$ according to (7). These equations and balances (5) determine equilibrium distribution of resources.

The following proposition is true for isolated systems.

Statement 1: *At the fixed distribution of money, the distribution of other resources in the equilibrium state corresponds to the maximum of total capitalization of economic agents subject to*

$$F(M_0,\overline{N}) = \sum_i \sum_j p_{ij}(\overline{N_{ij}}, M_j^0)\overline{N_{ij}} \to \max_{\overline{N_{ij}}}, \tag{25}$$

$$\sum_i \overline{N_{ij}} = N_i^0\ ,\ \ \overline{N_{ij}} \geq 0, \quad \text{i} = 1, \ldots, \text{k}; \quad \text{j} = 1, \ldots, \text{n}. \tag{26}$$

If the equilibrium stocks of all resources are positive then this maximum is equal to

$$F^*(M_0) = \sum_i \overline{p}_i N_i^0\ , \tag{27}$$

where $\overline{p}_i$ *is the equilibrium value of the i-th resource.*

MONETARY EXCHANGE MARKET

Let us consider a system containing a monetary exchange market. Each agent can change his currency for a basic resource. The exchange rate is considered to be constant and independent of the intensity of currency fluctuations. It means that the monetary exchange market is an economic reservoir. So, the value p_{0j} of the basic resource is constant to.

The existence of a monetary exchange market influences both the equilibrium state as well as the set of possible states of agents significantly.

Condition

$$p_{0j}(N, M) = k_j\ , \quad \text{j}= 1, \ldots, \text{n}, \tag{28}$$

connects stocks of basic and other resources and, therefore, restricts the states of each agent. The differential of capitalizations

$$dU_j = dM_j + \sum_i p_i(N_j, M_i) dN_{ij}\ ,$$

is a total differential on this set. If we express $M_j(N_j)$ from equation (28) and substitute this expression to p_{ij} then the following $n(n-1)/2$ relations hold for values $\tilde{p}_j = p_j(N, M(N))$:

$$\frac{\partial \tilde{p}_{ij}}{\partial N_{lj}} = \frac{\partial \tilde{p}_{lj}}{\partial N_{ij}}\ , \quad \text{i, l} = 1, \ldots, \text{k}; \ \ \text{j} = 1, \ldots, \text{n}. \tag{29}$$

Equilibrium in the closed system with a monetary exchange market is unique. It is determined by conditions (5), (7) with (28) additionally. The sum of reduced wealth functions

$$S = \sum_j S_j(\overline{N_j}, \overline{M_j}) \cdot k_j^{-1}\ , \tag{30}$$

with conditions (5) reaches the maximum in equilibrium state.

SYTEMS WITH AN INTERMEDIATE FIRM

Consider a system consisting of n economic agents. Exchange between agents is possible only through an intermediate firm. The initial state and wealth function of each agent is known. The problems are:

- to determine optimal time dependence of prices to maximize profit of the intermediate firm,
- to determine the maximal amount of basic resource which can be accumulated by the intermediate firm (profitability of the system).

First, we assume that the duration of the process is not restricted. For an isolated system the extracted resource is

$$E_\infty = \sum_{j=1}^{m} [M_j(0) - \overline{M_j}] \rightarrow \max\ , \tag{31}$$

where $\overline{M_j}$ is the amount of the basic resource of the j-th agent in the equilibrium state.

The profitability of the system will emerge from the equilibrium state. The value of the basic resource p_{0j} of each agent is positive. Hence the maximal amount of the basic resource extracted from the system in conditions of voluntariness corresponds to equations

$$S_j(\overline{N_j}, \overline{M_j}) = S_j[N_j(0), M_j(0)], \quad j = 1, \ldots, n. \tag{32}$$

These equations can be called conditions of reversibility of the process.

The values of each resource for each agent are equal in equilibrium:

$$p_{ij}(\overline{N_j}, \overline{M_j}) = \frac{\partial S_j}{\partial N_{ij}} \left(\frac{\partial S_j}{\partial M_j} \right)^{-1} = \overline{p}_i, \quad i = 1, \ldots, k; \ \ j = 1, \ldots, n. \tag{33}$$

Conditions (32), (33) together with the balance relations

$$\sum_{j=1}^{n} \overline{N_{ij}} = \sum_{j=1}^{n} N_{ij}(0), \quad i = 1, \ldots, k. \tag{34}$$

determine both the equilibrium state of the system and its profitability E_∞. So we can state that *equilibrium in the system with the intermediate firm is determined by the initial state of the system and does not depend on the kinetics of resource exchange*. There is, thus, a difference between economic systems with and without the intermediate firm.

In an economy with two agents and a scalar resource the states of the agents move along the corresponding level curves of the wealth functions. The distance between the equilibrium states is equal to the profitability of the system.

If the duration of the process is restricted then the value of the resource extracted by the intermediate firm is less than the profitability.

SYSTEMS WITH AN INTERMEDIATE FIRM CONTAINING AN ECONOMIC RESERVOIR

In a system with an economic reservoir equilibrium values are fixed for all agents and equal to the value of the reservoir. The number of equations here is $m \cdot n$

$$\overline{p}_{ij}(\overline{N_j}, \overline{M_j}) = p_{ri}, \quad i = 1, \ldots, k; \quad j = 1, \ldots, n. \tag{35}$$

Here p_{ri} is the value of the i-th resource of the economic reservoir.

If there is no intermediate firm the remaining n equations can be written using the kinetic equations of exchange between agents and between agents and the reservoir. If an intermediate firm exists conditions of exchange reversibility should be added.

$$S_j(N_{j0}, M_{j0}) = S_j(\overline{N_j}, \overline{M_j}) \quad j = 1, \ldots, n. \tag{36}$$

Let us use the Edgeworth box to depict the exchange of the scalar resource between two agents. The equilibrium state corresponds to the two points in the diagram. The values of the wealth functions of agents do not change for a system without an intermediate firm. But the distance between the points is composed of extracted profit and the amount of the resource obtained from the reservoir.

It can be shown that the maximal profit of the intermediate firm corresponds to the value p_r such that the second derivates of the basic resource with respect to N_j, calculated along calculated along the equipotential curves are equal in absolute value and opposite in sign.

CONCLUSION

Equilibrium conditions are obtained for different structures of a system. In some cases, the equilibrium state depends on the kinetics of resource exchange, in others on the initial state alone.

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MATEMATIČKI MODELI I STANJE RAVNOTEŽE U IREVERZIBILNOJ MIKROEKONOMIJI

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SAŽETAK

Razmatran je skup ravnotežnih stanja sustava koji se sastoji od ekonomskih agenata, ekonomskih rezervoara i tvrtki. Korištene su metode ireverzibilne mikroekonomije. Pokazano je da izravna kupoprodaja vodi na ravnotežno stanje koje ovisi o koeficijentima funkcija ponude i potražnje. Za postizanje jedinstvenog ravnotežnog stanja potrebna je ili kupnja novcem, ili tvrtka za međutransakcije.

KLJUČNE RIJEČI

ireverzibilna mikroekonomija, matematički modeli, termodinamika